Rare-Earth Mixed Crystals for Fast Optical Quantum Computers

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state	Pr^{3+}	Er^{3+}	Tm^{3+}	Tm^{3+}	Tm^{3+}	Tm^{3+}
$\|0\rangle$	$\|^1G_4\rangle$	$\|^4I_{9/2}\rangle$	$\|^3F_4\rangle$	$\|^3H_4\rangle$	$\|^3F_4\rangle$	$\|^3F_4\rangle$
$\|1\rangle$	$\|^3P_0\rangle$	$\|^4S_{3/2}\rangle$	$\|^1D_2\rangle$	$\|^1D_2\rangle$	$\|^1D_2\rangle$	$\|^3H_4\rangle$
$\|1'\rangle$	$\|^3H_4\rangle$	$\|^4I_{15/2}\rangle$	$\|^3H_6\rangle$	$\|^1I_6\rangle$	$\|^1I_6\rangle$	$\|^1I_6\rangle$

Examples of RE ions and levels which can be used for fast quantum computers

Abstract

- The possibility of using rare-earth (RE) mixed crystals as physical systems for creating fast quantum computers with nanosecond sampling time is discussed. Various electronic 4f-states of RE ions are considered as qubit levels with optical frequencies. A high computational speed can be achieved due to large inhomogeneous broadening of the frequencies of electronic transitions in these systems. Another advantage of these systems is the weak electron-phonon interaction, which allows to obtain qubits with a long decoherence time.
- The advantage is also a large number of 4f electronic states in RE ions, which have strongly different values of the elements of the matrices ***U*** of Judd-Ofelt theory, which describes the properties of these ions. Such a variety of these matrix elements makes it possible to find systems in which, along with long-living weakly interacting states suitable for qubits, there are also states interacting strongly enough to carry out the conditional gate operations. In this case, the electronic 4f-states of RE ions with small values of the diagonal elements of $\boldsymbol{U}^{(2)}$ matrix are suitable to use as levels of qubits (**|0>** and **|1>**). CNOT and other conditional gate operations are performed by temporarily placing the RE ion in the auxiliary 4f-state (**|1'>**) with a large diagonal element of $\boldsymbol{U}^{(2)}$ matrix, allowing to implement the required Stark blockade.
- It is found that the main interaction responsible for this blockade (for moderate distances between RE ions) is the quadrupole-quadrupole interaction. This result contradicts the generally accepted opinion that the main interaction responsible for the Stark blockade is the dipole-dipole interaction (which is why the term dipole blockade was commonly used to refer to this phenomenon). A number of specific systems are proposed that are promising for the implementation of fast optical quantum computers.

Advantages of rare-earth (RE) mixed crystals as physical systems for fast quantum computers

- A high computational speed of optical quantum computers in RE mixed crystals $La_{1-x}Y_xF_3$, $(SrF_2)_x(CaF_2)_{1-x}$, and similar can be achieved for RE ions due to **(a)** possibility to use **optical frequency qubits** and **(b)** due to **large inhomogeneous broadening** of the frequencies of electronic transitions ~1THz.
- The weak electron-phonon interaction allows to obtain qubits with a long decoherence time. The latter is supported by the small number of tunnelling systems and other low-energy excitations.
- A large number of 4f electronic states with strongly different Judd-Ofelt parameters allows one to find systems, in which weakly interacting states are suitable for qubits. Along with these, there are also states that interact strongly enough to implement the required Stark blockade and perform the conditional gate operations.

Main properties

- Fast quantum computers operate with short (ns) light pulses having remarkable spectral width. The large inhomogeneous width ~1THz of zero-phonon transitions in $La_{1-x}Y_xF_3$, $(SrF_2)_x(CaF_2)_{1-x}$, allows one to selectively address large number of different doping RE ions (qubits).
- The working sample can be prepared by coating the substrate with crystalline nanoparticles. The use of nanocrystals allows us to work with finite number of qubits.
- The mixed crystals under consideration can be used with arbitrary concentration of dopants. This allows one to achieve strong interaction between RE ions and, thus, qubits.
- The main parameters determining the strength of the interaction of ions are the diagonal elements of the Judd-Ofelt matrices $\boldsymbol{U}^{(2)}$.
- The working interaction of RE ions is the quadrupole-quadrupole interaction. This result contradicts the generally accepted opinion that it is the dipole-dipole interaction.
- A number of specific systems are **proposed**, that are promising for the implementation of fast optical quantum computers.

Light pulses

To perform gate operations one needs to use pi-pulses of light with the field strength

$$|E| = 2\pi\Gamma_L\sqrt{\hbar k^3/3\gamma_0}$$

where $\boldsymbol{\Gamma_L}$ is the spectral width of the pulse, $\boldsymbol{\gamma_0}$ is the rate of radiative decay, ***k*** is wave vector of optical transition in qubit. For nano-pulses one gets **E**~10^4 V/cm, which corresponds to power **W**~10^6W/cm^2, which can be easily achieved with modern laser systems.

CNOT gates

To perform **control gate operations**, the corresponding RE ions must interact with each other. To this end one can use the Stark blockade.

We have found that for **RE ions the main interaction leading to Stark blockade is the quadrupole-quadrupole interaction**. The energy of this interaction is described by the formula

$$\delta^{(q)} \sim \alpha\omega_0\left(U_{01}^{(2)}\Big/U_{00}^{(2)}\right)^2 r_0^4\Big/\varepsilon_0 k R^5$$

where **α**~0.1, $\mathbf{r_0}$ = 1Å is the size of the 4f-state, **R** is the distance between RE ions, $\boldsymbol{\varepsilon_0}$ is dielectric constant, $\boldsymbol{U^{(2)}}$ is the Judd-Ofelt matrix, subscripts denote the initial (0) and final (1) levels. For large working nondiagonal elements of matrix $\boldsymbol{U^{(2)}}$ one gets for RE ions at intermediate distances R~20Å the shift δ(q)~10GHz, which is sufficient for Stark blockade.

States of RE ions suitable for fast quantum computers

For fast quantum computers one can use:

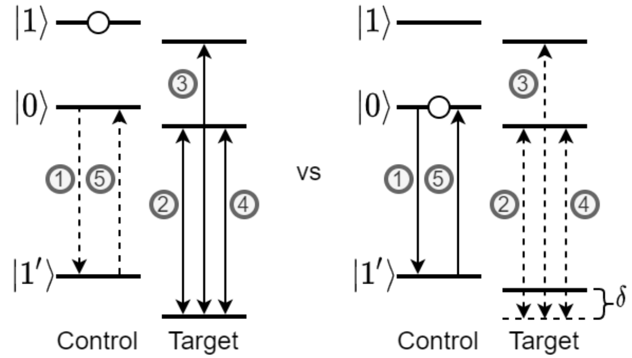
a) 4f-levels with small diagonal $\boldsymbol{U^{(2)}}$ as qubit levels;
b) 4f-levels with large diagonal $\boldsymbol{U^{(2)}}$ as auxiliary levels for CNOT gate operations.

Examples of RE ions and levels which can be used for fast quantum computers

state	Pr^{3+}	Er^{3+}	Tm^{3+}	Tm^{3+}	Tm^{3+}	Tm^{3+}
$\|0\rangle$	$\|{}^1G_4\rangle$	$\|{}^4I_{9/2}\rangle$	$\|{}^3F_4\rangle$	$\|{}^3H_4\rangle$	$\|{}^3F_4\rangle$	$\|{}^3F_4\rangle$
$\|1\rangle$	$\|{}^3P_0\rangle$	$\|{}^4S_{3/2}\rangle$	$\|{}^1D_2\rangle$	$\|{}^1D_2\rangle$	$\|{}^1D_2\rangle$	$\|{}^3H_4\rangle$
$\|1'\rangle$	$\|{}^3H_4\rangle$	$\|{}^4I_{15/2}\rangle$	$\|{}^3H_6\rangle$	$\|{}^1I_6\rangle$	$\|{}^1I_6\rangle$	$\|{}^1I_6\rangle$

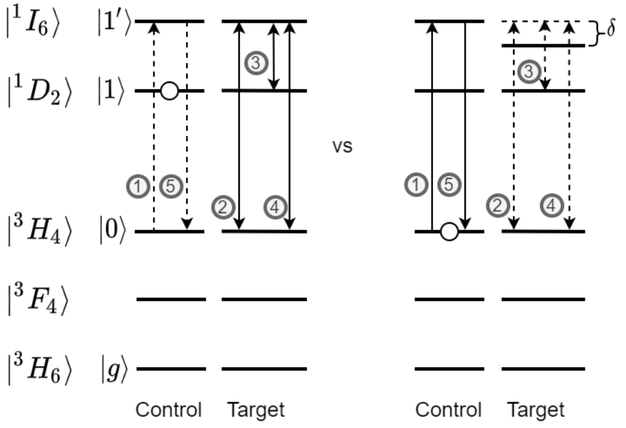
States of RE ions suitable for fast quantum computers

The scheme of the CNOT gate operations with Pr^{3+} and Er^{3+} ions



NOT performed NOT failed

The scheme of the CNOT gate operations with Tm^{3+} ions



NOT performed NOT failed

Frequencies, lifetimes and elements of Judd-Ofelt matrices for Pr^{3+} and Er^{3+} levels

ion	state	level	E, cm^{-1}	τ, μs	$\lvert U^{(2)}\rvert^2$ $\lvert 0\rangle$			$\lvert U^{(2)}\rvert^2$ $\lvert 1\rangle$			$\lvert U^{(2)}\rvert^2$ $\lvert 1'\rangle$		
Pr^{3+}	$\lvert 0\rangle$	$\lvert ^1G_4\rangle$	9640	14	0	0	0	0	.056	0	.012	.072	.027
	$\lvert 1\rangle$	$\lvert ^3P_0\rangle$	20469	55	0	.056	0	0	0	0	0	.173	0
	$\lvert 1'\rangle$	$\lvert ^3H_4\rangle$	0	∞	.012	.072	.027	0	.173	0	.779	0	0
Er^{3+}	$\lvert 0\rangle$	$\lvert ^4I_{9/2}\rangle$	12272	133	.002	0	0	0	.079	.254	0	.173	.010
	$\lvert 1\rangle$	$\lvert ^4S_{3/2}\rangle$	18353	923	0	.079	.254	.037	0	0	0	0	.223
	$\lvert 1'\rangle$	$\lvert ^4I_{15/2}\rangle$	0	∞	0	.173	.010	0	0	.223	.247	0	0

Frequencies, lifetimes and elements of Judd-Ofelt matrices for Tm^{3+} levels

ion	level	E, cm^{-1}	τ, μs	$U^{(2)}$ $U^{(4)}$ $U^{(6)}$ $\lvert ^3H_6\rangle$	$\lvert ^3F_4\rangle$	$\lvert ^3H_4\rangle$	$\lvert ^1D_2\rangle$	$\lvert ^1I_6\rangle$
Tm^{3+}	$\lvert ^3H_6\rangle$	0	∞	1.25 .691 .772	.537 .726 .238	**.237** .109 .595	0 **.316** .093	.011 .039 .001
	$\lvert ^3F_4\rangle$	5610	18050	**.537** .726 .238	.001 .409 .269	.129 .130 .206	**.575** .096 .023	0 .108 0
	$\lvert ^3H_4\rangle$	12518	2890	**.237** .109 .595	.129 .130 .206	.268 1.62 .583	**.127** .012 .228	.066 **.305** .097
	$\lvert ^1D_2\rangle$	27830	70	0 **.316** .093	**.575** .096 .023	**.127** .012 .228	.197 .004 0	0 .051 **.838**
	$\lvert ^1I_6\rangle$	34684	300	.011 .039 .001	0 **.108** 0	.066 **.305** .097	0 .051 **.838**	4.88 1.58 .119

Conclusions

- Mixed crystals $La_{1-x}Y_xF_3$, $(SrF_2)_x(CaF_2)_{1-x}$, and similar, doped with rare-earth ions Tm^{3+} and others, can serve as physical system for the fabrication of **fast optical quantum computers**. The qubits correspond to the different 4f-levels and they have an optical frequency.
- The weak electron-phonon interaction allows to obtain qubits with a long decoherence time.
- Large inhomogeneous width of zero-phonon transitions makes it possible operate with high speed ~GHz.
- The main parameters determing the strength of the interaction of the RE ions are the non-diagonal elements of the Judd-Ofelt matrices $\boldsymbol{U}^{(2)}$.
- The strong differences of Judd-Ofelt parameters of different 4f-states make it possible to find systems in which, along weakly interacting states, suitable for qubits, there are also states interacting strongly enough to implement the Stark blockade and to carry out the CNOT operations.
- The main interaction responsible for Stark blockade is the quadrupole-quadrupole interaction.
- The working sample can be prepared by coating the substrate with **crystalline nanoparticles**. The use of nanocrystals allows one to work with finite number of qubits.