Analytical view on N bodies interacting with quantum cavity in tight-binding model

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Wannier qubit in the chain of coupled q-dots

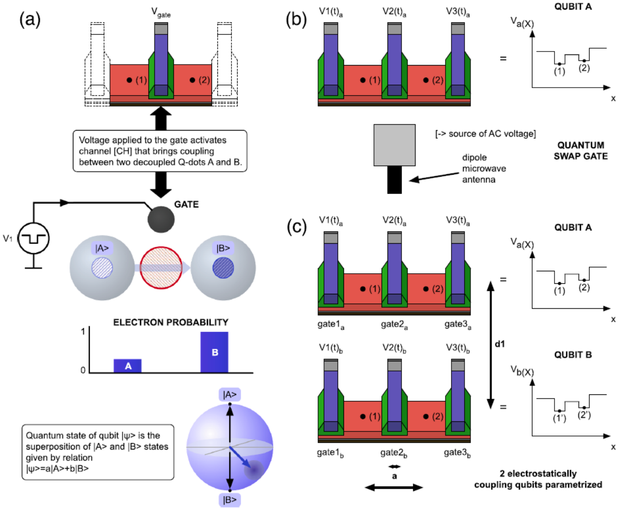

Figure: Basic concept of Wannier (position-based qubit) as from [Cryogenics 2020, Pomorski et al.].

Q-communication

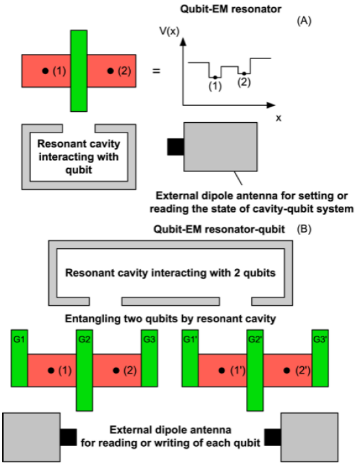

Figure: Position based qubit in RF field (A) and position based qubits placed at high distance interlinked by waveguide (B) [Pomorski et al. , AIP Conference Proceedings 2020]. Physical states of qubits are controlled by voltages applied to the gates G1, G2, G3 and G1', G2', G3'.

Jaynes-Cummings tight-binding Hamiltonian

$$H = H_{qubit} + H_{QEC} + H_{qubit-QEC}, \hat{H}_t \left|\psi\right\rangle_t = i\hbar \frac{d}{dt} \left|\psi\right\rangle_t . \qquad (1)$$

It leads to the following Hamiltonian ($H_{qubit-QEC} = \hat{E} \cdot \hat{d}$) given as

$$H = I_{qed} \times H_{qubit} + H_{qed} \times I_{qubit} + H_{qed-qubit} =$$
$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \times \begin{pmatrix} E_{p1} & t_s \\ t_s & E_{p2} \end{pmatrix} + \begin{pmatrix} E_{cav1} & 0 \\ 0 & E_{cav2} \end{pmatrix} \times \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} +$$
$$+(x_2 - x_1)e \begin{pmatrix} E_{f1}(t) & 0 \\ 0 & E_{f2}(t) \end{pmatrix} \times \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} =$$
$$= \begin{pmatrix} \hat{H}_{eff1}(t) & \hat{0}_{2\times 2} \\ \hat{0}_{2\times 2} & \hat{H}_{eff2}(t) \end{pmatrix},$$

$$\hat{H}_{eff1}(t) = \begin{pmatrix} E_{c1} - E_{f1q}(t) + E_{p1}(t) & t_s(t) \\ t_s^*(t) & E_{c1} + E_{f1q} + E_{p2}(t) \end{pmatrix},$$
$$\hat{H}_{eff2}(t) = \begin{pmatrix} E_{c2} - E_{f2q}(t) + E_{p1}(t) & t_s(t) \\ t_s^*(t) & E_{c2} + E_{f2q} + E_{p2}(t) \end{pmatrix}, \qquad (2)$$

with $E_{pp1}(t) = E_{c1} - E_{f1q}(t) + E_{p1}(t)$, $E_{pp2}(t) = E_{c1} + E_{f1q}(t) + E_{p2}(t)$,
$t_s(t) = t_{sr} + it_{si}, E_{f1}(t) = E_{f1q}\sqrt{\frac{2}{\varepsilon}\hbar\omega} cos(\omega t), E_{f2q} \propto sin(2\omega t).$

$$|\psi(t)>= \frac{\sqrt{p_{E1}(t)}e^{i\phi_{E1(t)}}}{\sqrt{(-2E_{f1}+E_{p1}-E_{p2}-\sqrt{(2E_{f1}-E_{p1}+E_{p2})^2+4|t_s|^2))^2+4|t_s|^2}}\times$$

$$\times\begin{pmatrix}(-2E_{f1}+E_{p1}-E_{p2}-\sqrt{(2E_{f1}-E_{p1}+E_{p2})^2+4|t_s|^2)})e^{i\alpha}\\ 2|t_s|\\ 0\\ 0\end{pmatrix}+$$

$$\frac{\sqrt{p_{E2}(t)}e^{i\phi_{E2(t)}}}{\sqrt{(-2E_{f1}+E_{p1}-E_{p2}+\sqrt{(2E_{f1}-E_{p1}+E_{p2})^2+4|t_s|^2))^2+4|t_s|^2}}\times$$

$$\times\begin{pmatrix}(-2E_{f1}+E_{p1}-E_{p2}+\sqrt{(2E_{f1}-E_{p1}+E_{p2})^2+4|t_s|^2)})e^{i\alpha}\\ 2|t_s|\\ 0\\ 0\end{pmatrix}+$$

$$+\frac{\sqrt{p_{E3}(t)}e^{i\phi_{E3(t)}}}{\sqrt{(-2E_{f2}+E_{p1}-E_{p2}-\sqrt{(2E_{f1}-E_{p1}+E_{p2})^2+4|t_s|^2)})^2+4|t_s|^2}}\times$$

$$\times\begin{pmatrix}0\\ 0\\ (-2E_{f1}+E_{p1}-E_{p2}-\sqrt{(2E_{f1}-E_{p1}+E_{p2})^2+4|t_s|^2)})e^{i\alpha}\\ 2|t_s|\end{pmatrix}+$$

$$+\frac{\sqrt{p_{E4}(t)}e^{i\phi_{E4(t)}}}{\sqrt{(-2E_{f1}+E_{p1}-E_{p2}+\sqrt{(2E_{f1}-E_{p1}+E_{p2})^2+4|t_s|^2))^2+4|t_s|^2}}\times$$

$$\begin{pmatrix}0\\ 0\\ (-2E_{f1}+E_{p1}-E_{p2}+\sqrt{(2E_{f1}-E_{p1}+E_{p2})^2+4|t_s|^2)})e^{i\alpha}\\ 2|t_s|\end{pmatrix}$$

Explicit formulas for q-state evolution, entanglement entropy, switching between energy levels controlled by electrostatic voltages

$$
|\psi(t)>= e^{(\int_{t_0}^{t} \frac{1}{\hbar i}\hat{H}(t')dt')}|\psi(t_0)>=
$$
$$
= \begin{pmatrix} e^{\int_{t_0}^{t} \frac{1}{\hbar i}\hat{H}_{eff1}(t')dt'} & \hat{0}_{2\times 2} \\ \hat{0}_{2\times 2} & e^{\int_{t_0}^{t} \frac{1}{\hbar i}\hat{H}_{eff2}(t')dt'} \end{pmatrix} |\psi(t_0)>=
$$
$$
= \begin{pmatrix} U_{1,1}(t,t_0) & U_{1,2}(t,t_0) & 0 & 0 \\ U_{2,1}(t,t_0) & U_{2,2}(t,t_0) & 0 & 0 \\ 0 & 0 & U_{3,3}(t,t_0) & U_{3,4}(t,t_0) \\ 0 & 0 & U_{4,3}(t,t_0) & U_{4,4}(t,t_0) \end{pmatrix} \tag{3}
$$

Explicit formulas for q-state evolution for any type of Wannier qubit interacting with 2-level q-cavity in tight-binding Hamiltonian ($\int \to \int_{t0}^{t}$)...

$$U_{11}(t,t_0) = e^{\frac{i(\int \mathrm{Epp1}(t')\,dt' + \int \mathrm{Epp2}(t')\,dt')}{2\hbar}} \times$$

$$\left[\frac{i(\int \mathrm{Epp1}(t')\,dt' - \int \mathrm{Epp2}(t')\,dt')\sin\left(\frac{\sqrt{(\int \mathrm{Epp1}(t')\,dt' - \int \mathrm{Epp2}(t')\,dt')^2 + 4(\int(\mathrm{tsr}(t') - i\mathrm{tsi}(t'))\,dt')\int(\mathrm{tsr}(t') + i\mathrm{tsi}(t'))\,dt'}}{2\hbar}\right)}{\sqrt{(\int \mathrm{Epp1}(t')\,dt' - \int \mathrm{Epp2}(t')\,dt')^2 + 4(\int(\mathrm{tsr}(t') - i\mathrm{tsi}(t'))\,dt')\int(\mathrm{tsr}(t') + i\mathrm{tsi}(t'))\,dt'}} + \right.$$

$$\left. + \cos\left(\frac{\sqrt{(\int \mathrm{Epp1}(t')\,dt' - \int \mathrm{Epp2}(t')\,dt')^2 + 4(\int(\mathrm{tsr}(t') - i\mathrm{tsi}(t'))\,dt')\int(\mathrm{tsr}(t') + i\mathrm{tsi}(t'))\,dt'}}{2\hbar}\right)\right],$$

$$U_{12}(t,t_0) =$$

$$\frac{2i(\int(\mathrm{tsr}(t') + i\mathrm{tsi}(t'))\,dt')e^{\frac{i(\int \mathrm{Epp1}(t')\,dt' + \int \mathrm{Epp2}(t')\,dt')}{2\hbar}}\sin\left(\frac{\sqrt{(\int \mathrm{Epp1}(t')\,dt - \int \mathrm{Epp2}(t')\,dt')^2 + 4(\int(\mathrm{tsr}(t') - i\mathrm{tsi}(t'))\,dt')\int(\mathrm{tsr}(t') + i\mathrm{tsi}(t'))\,dt'}}{2\hbar}\right)}{\sqrt{(\int \mathrm{Epp1}(t')\,dt' - \int \mathrm{Epp2}(t')\,dt')^2 + 4(\int(\mathrm{tsr}(t') - i\mathrm{tsi}(t'))\,dt')\int(\mathrm{tsr}(t') + i\mathrm{tsi}(t'))\,dt'}}$$

$$U_{22}(t,t_0) =$$

$$e^{\frac{i(\int \mathrm{Epp1}(t')\,dt' + \int \mathrm{Epp2}(t)\,dt')}{2\hbar}}\left(\cos\left(\frac{\sqrt{(\int \mathrm{Epp1}(t')\,dt' - \int \mathrm{Epp2}(t')\,dt')^2 + 4(\int(\mathrm{tsr}(t') - i\mathrm{tsi}(t'))\,dt')\int(\mathrm{tsr}(t') + i\mathrm{tsi}(t'))\,dt'}}{2\hbar}\right)\right.$$

$$\left. - \frac{i(\int \mathrm{Epp1}(t')\,dt' - \int \mathrm{Epp2}(t')\,dt')\sin\left(\frac{\sqrt{(\int \mathrm{Epp1}(t')\,dt' - \int \mathrm{Epp2}(t)\,dt')^2 + 4(\int(\mathrm{tsr}(t') - i\mathrm{tsi}(t'))\,dt')\int(\mathrm{tsr}(t') + i\mathrm{tsi}(t'))\,dt'}}{2\hbar}\right)}{\sqrt{(\int \mathrm{Epp1}(t')\,dt' - \int \mathrm{Epp2}(t')\,dt')^2 + 4(\int(\mathrm{tsr}(t') - i\mathrm{tsi}(t'))\,dt')\int(\mathrm{tsr}(t') + i\mathrm{tsi}(t'))\,dt'}}\right)$$

Literature

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