Mapping graph state orbits under local complementation

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1. Graph states & local complementation
Graph states have a one-to-one correspondence with a mathematical graph $G = (V, E)$:
\[ (|G\rangle = \prod_{(i,j) \in E} \mathbb{C}z_{ij}^{+}) \]

2. Orbits of group actions (Cayley graphs)
Repeated applications of local complementation defines entanglement classes of locally equivalent graph states:

3. Isomorphic graph states
- We can also consider isomorphic graphs to be equal.
- This greatly simplifies the orbits.
- We computed the first 576 orbits when considering isomorphic graphs equal (up to 10 qubits), and the first 146 otherwise (up to 9 qubits).
- We provide a program to generate the orbit of any graph, with out data set and a program to draw these figures (online).

4. Inferring graph properties from their orbits
- 'Star' graph states (GHZ states) have only the other star isomorphisms and the fully-connected graph states in their orbit, which is itself star (see §1).
- Orbits with no self-loops do not contain any graph states which have a vertex of degree 1 (a leaf).
- Orbits which are trees only contain graphs in which all vertices are at most distance two separated.

5. Exploiting symmetry
Considering isomorphic graphs equal, we only local complement asymmetric vertices.

6. Orbits are often identical!
These graph states all have isomorphic orbits:

7. Correlating orbits with quantum properties
- Orbit diameter and orbit chromatic number correlate strongly with Schmidt measure.
  - For both types of orbit: considering isomorphic graphs equal (C), and not (L).
- Chromatic index correlates with Schmidt measure strongly for C and moderately for L.
- Graph state chromatic index does correlate with Schmidt measure.
  - This is the number of CZ time steps needed to prepare the state.
  - The min no edges of a graph state in the orbit does correlate with Schmidt measure.
  - Local complementation can reduce the number of CZ gates needed.
- The Schmidt measure correlates with the graph rank width.
  - Important in the complexity of graph state algorithms.

8. Applications & Outlook
- Local complementation can compile different quantum protocols with the same resource state.
- Basic change preserves the 'standard' language of measurement-based protocols.
- Application in optimising a resource for error correction.
- Optimal resource state preparation by interspersing CZ with local complementation.
- Are there more connections between graph properties and quantum properties?
- What can we learn from the symmetry of an orbit?

References