Mapping graph state orbits under local complementation

Jeremy C. Adcock[1,2], Sam Morley-Short[2], Axel Dahlberg[3], Joshua W. Silverstone[4]

[1]DTU Fotonik, Denmark Technical University, Building 343, Ørsteds Pl., 2800 Kgs. Lyngby, Denmark [2]PsiQuantum, 700 Hansen Way, Palo Alto, CA 94304, United States
[3]QuTech - TU Delft, Lorentzweg 1, 2628CJ Delft, The Netherlands [4]QETLabs, University of Bristol, Merchant Venturers Building, Woodland Road BS8 1UB, UK

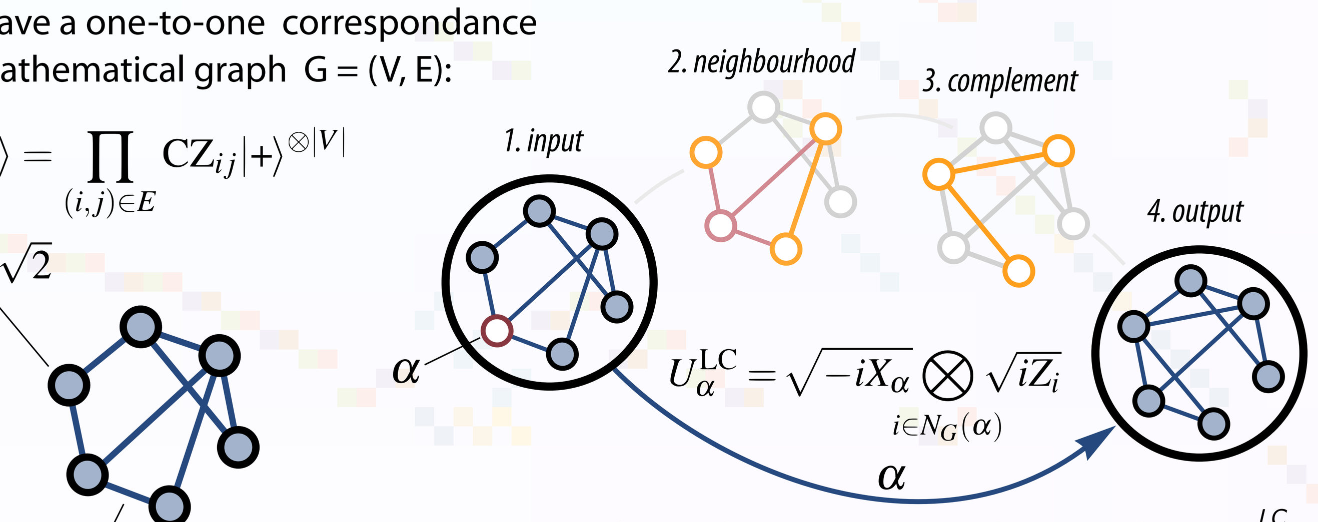
1. Graph states & local complementation

Graph states have a one-to-one correspondance with a mathematical graph G = (V, E):

$$|G\rangle = \prod_{(i,j)\in E} CZ_{ij}|+\rangle^{\otimes|V|}$$

$$|+\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$$

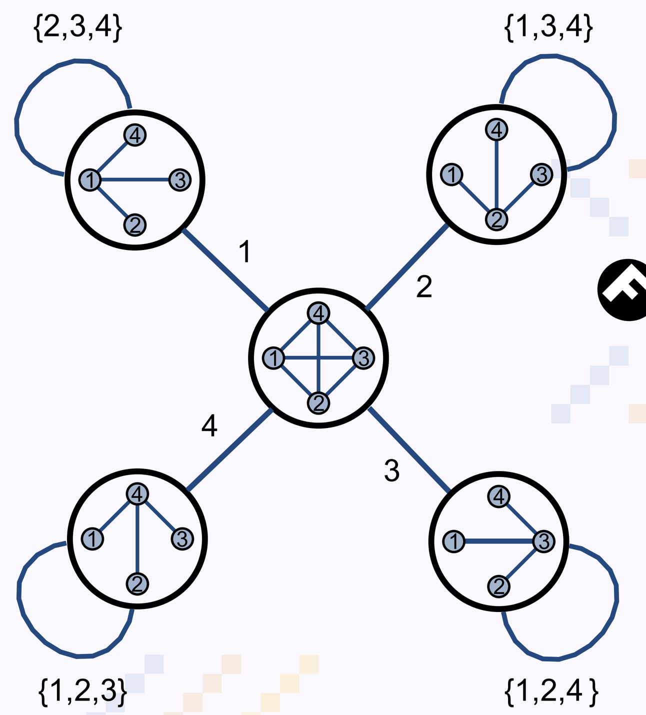
$$CZ = |00\rangle\langle 00| + |01\rangle\langle 01| + |10\rangle\langle 10| - |11\rangle\langle 11|$$



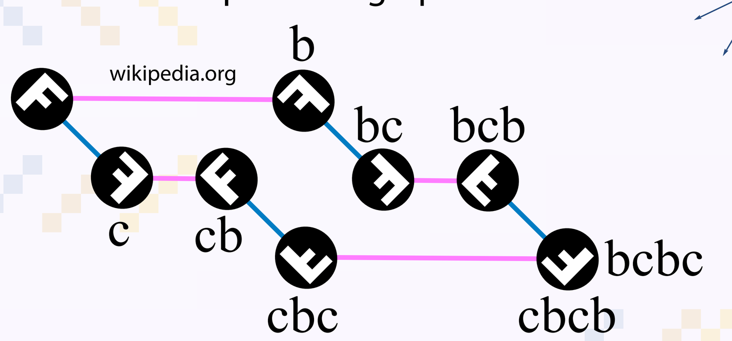
$$U_\alpha^{LC} = \sqrt{-iX_\alpha}\bigotimes_{i\in N_G(\alpha)}\sqrt{iZ_i}$$

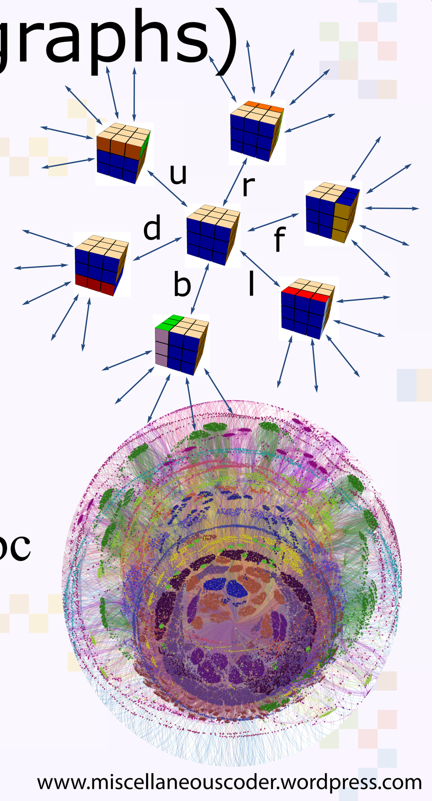
Local complementation, implemented with U^{LC}, traverses all locally equivalent graph states

2. Orbits of group actions (Cayley graphs)



Repeated applications of local complementation defines entanglement classes of locally equivalent graph states

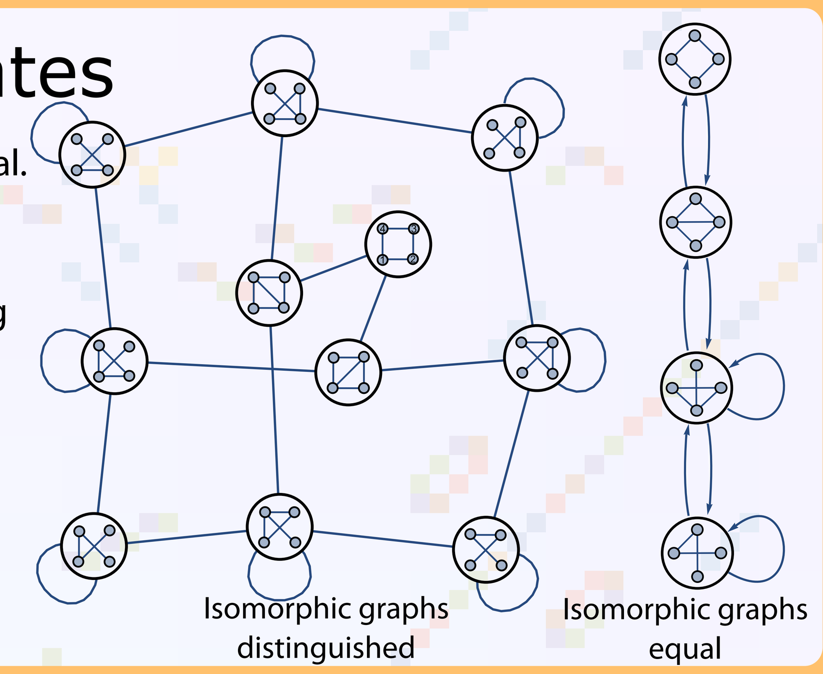




This produces orbits, similar to the reflection/rotation orbit (above) and rubiks cube group (right)

3. Isomorphic graph states

- We can also consider isomorphic graphs to be equal.
- This greatly simplifies the orbits
- We computed the first 576 orbits when considering isomorphic graphs equal (up to 10 qubits), and the first 146 otherwise (up to 9 qubits).
- We provide a program to generate the orbit of any graph, with out data set and a program to draw these figures (online).[5,6]

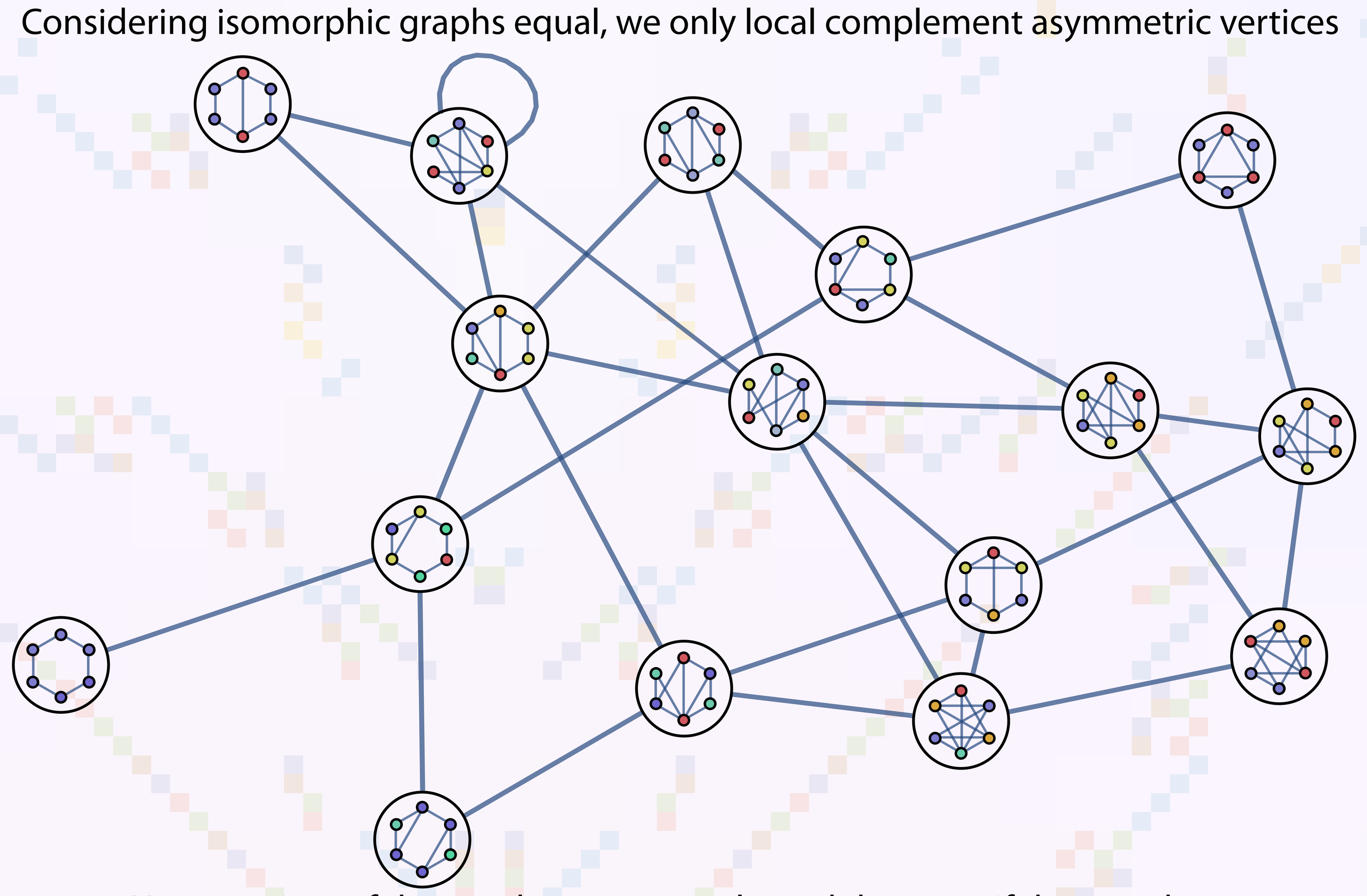


4. Infering graph properties from their orbits

- 'Star' graph states (GHZ states) have only the other star isomorphisms and the fully-connected graph states in their orbit, which is itself star (see §1.).
- Orbits with no self-loops do not contain any graph states which have a vertex of degree 1 (a leaf).
- Orbits which are trees only contain graphs in which all vertices are at most distance two seperated.

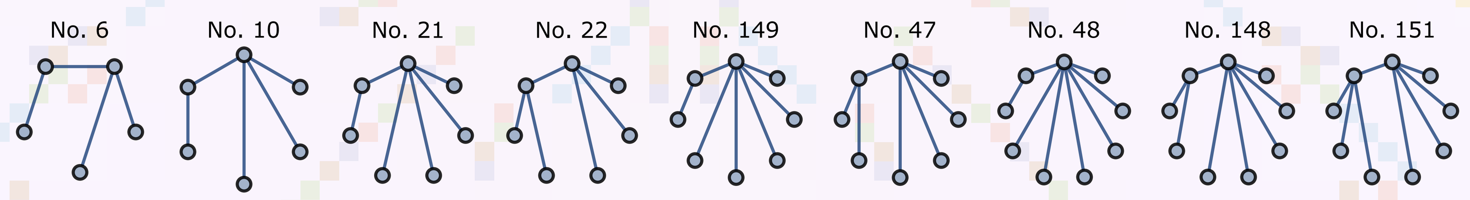
5. Exploiting symmetry

Considering isomorphic graphs equal, we only local complement asymmetric vertices

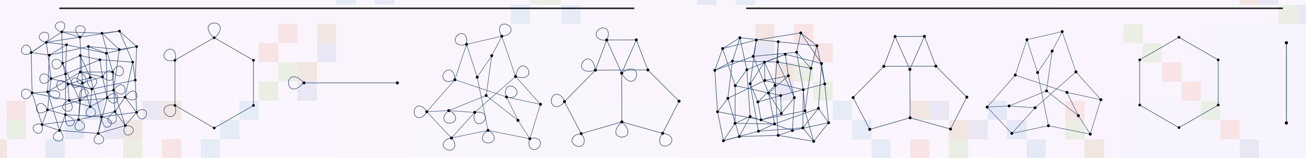


Here, vertices of the graph states are coloured the same if they produce the same graph state under local complementation

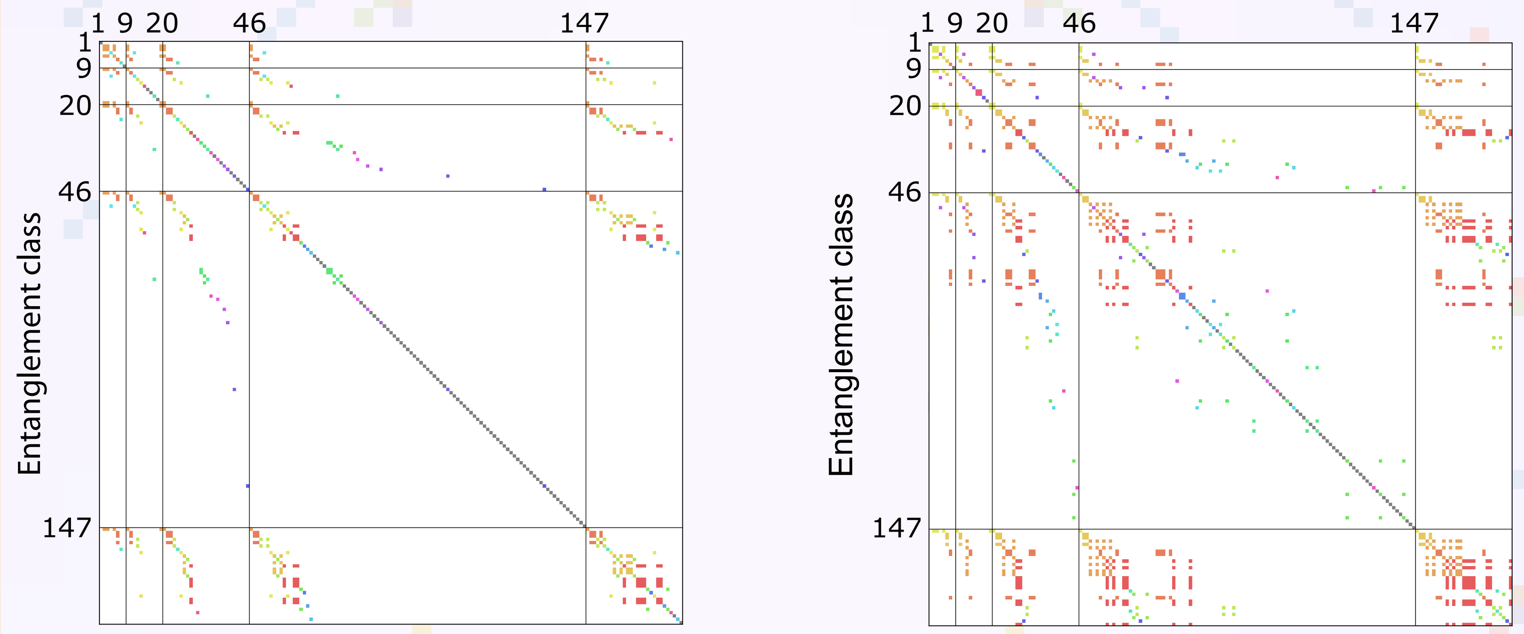
6. Orbits are often identical!



These graph states all have isomorphic orbits!



	Loops included					Loops removed				
Orbit										
Size of orbit	44	6	2	16	10	44	10	16	6	2
No. iso. orbits	10	9	8	7	6	17	15	13	9	8
Legend										



Here, $M_{ij} = 1$ if graph entanglement class i and j have isomorphic orbits

7. Correlating orbits with quantum properties

- Orbit diameter and orbit chromatic number correlate strongly with Schmidt measure.
  - For both types of orbit: considering isomorphic graphs equal (C), and not (L).
- Chromatic index correlates with Schmidt measure strongly for C and moderately for L.
- Graph state chromatix index does correlate with Schmidt measure.
  - This is the number of CZ time steps needed to prepare the state.
- The min no. edges of a graph state in the orbit does correlate with Schmidt measure.
  - This is the total number of CZs needed to prepare the state.
  - Local complementation can reduce the number of CZ gates required[4].
- The Schmidt measure correlates with the graph rank width.
  - Important in the complexity of graph state algorithms.

Object	Property x	y	Correlation $-1 < r(x,y) < 1$
C orbit	diameter	orbit size	0.62 ± 0.03
C orbit	diameter	Schmidt measure	0.77 ± 0.02
C orbit	chromatic number	Schmidt measure	0.67 ± 0.02
C orbit	chromatic index	Schmidt measure	0.81 ± 0.04
L orbit	diameter	orbit size	0.060 ± 0.05
L orbit	diameter	Schmidt measure	0.93 ± 0.02
L orbit	chromatic index	Schmidt measure	0.70 ± 0.05
L orbit	chromatic number	Schmidt measure	0.44 ± 0.11
graph state	Schmidt measure	min. no edges	0.78 ± 0.02
graph state	Schmidt measure	chromatic index	−0.17 ± 0.02
graph state	Schmidt measure	rank width	0.62 ± 0.03

C orbit: Orbit with isomorphic graphs equal ***orbit size***: number of graph states in the orbit
L orbit: Orbit without isomorphic graphs equal ***Schmidt measure***: entanglement metric
diameter: The smallest number of local complementations needed to go from qubit i to qubit j
chromatic number: smallest number of colours needed to colour the vertices of a graph so that no vertices of the same colour are adjacent
chromatic index: smallest number of colours needed to colour the edges of a graph so that no edges of the same colour are adjacent

8. Applications & Outlook

- Local complementation can compile different quantum protocols with the same resource state.
  - Basis change preserves the 'standard' language of measurement-based protocols.
  - Application in optimising a resource for error correction.
- Optimal resource state preparation by interspersing CZ with local complementaiton.[4]
- Are there more connections between graph properties and quantum properties?
- What can we learn from the symmetry of an orbit?

References

[1] Jeremy C. Adcock et al. "Mapping graph state orbits under local complementation" ***Quantum*** 4 (2020): 305
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[5] Graph orbit generator: https://doi.org/10.5281/zenodo.2582616
[6] Orbit data: https://doi.org/10.5281/zenodo.2582616