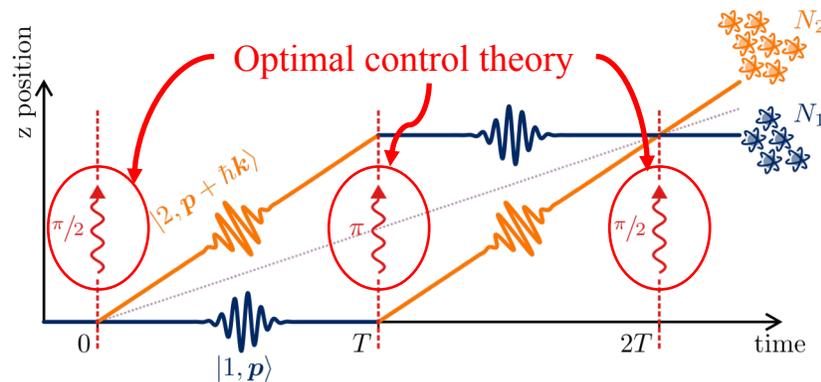


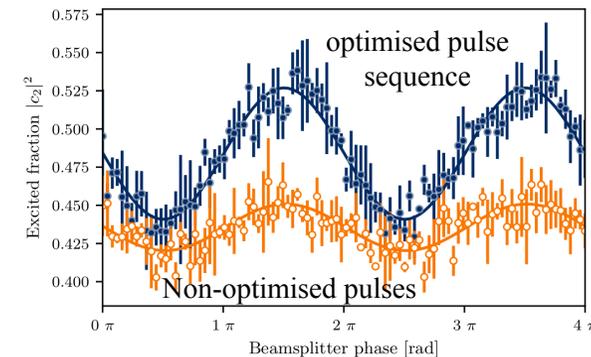
# Optimal control methods for enhancing the sensitivity and robustness of atom interferometric sensors

- The contrast and sensitivity of **atom interferometers** are limited by the fidelity of the beamsplitter and mirror light pulses, which can be sensitive to experimental inhomogeneities e.g. **variations in atomic velocity and laser intensity**
- We use **optimal control theory** to design tailored **Raman pulses** - optimised sequences of laser phase shifts:
  - **99.8(3)% state transfer** in a 35  $\mu\text{K}$  ensemble of  $^{85}\text{Rb}$
  - **threefold improvement in the visibility** of interferometer fringes at 100  $\mu\text{K}$
  - Optimised pulse sequences for **large momentum transfer** interferometers provide significant improvements in measurement scale-factor

Spacetime diagram of atom interferometer:  $\pi/2$  and  $\pi$  **Raman pulses** split, redirect, and interfere atomic wavepackets.



Apply optimised pulse sequence to increase interference contrast



# 1. Contrast loss in atom interferometry

## Problem:

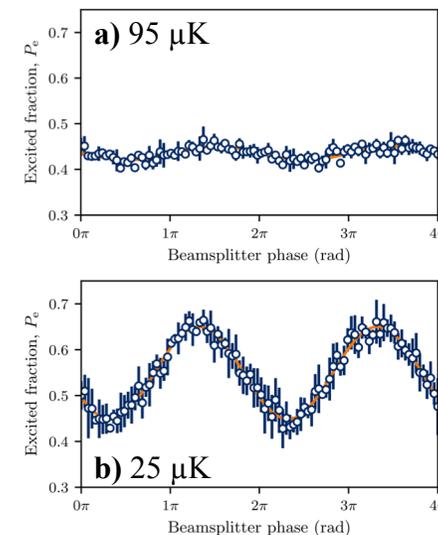
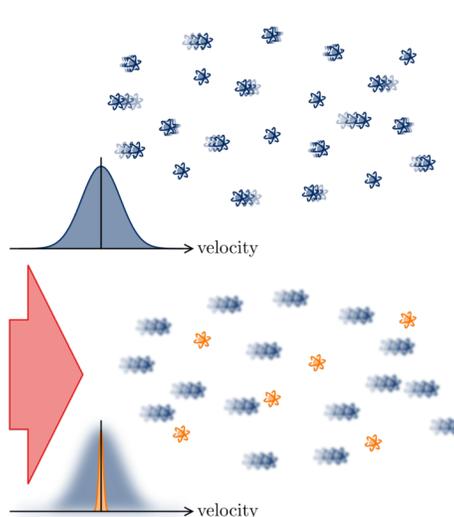
Atomic velocity spread causes variations in **detuning**

## Fix:

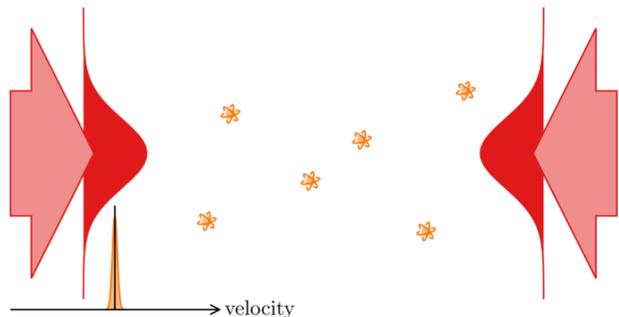
**Filter** atomic sample to get narrower velocity spread

## But:

**Fewer atoms**, reduced signal-to-noise ratio and higher experimental complexity

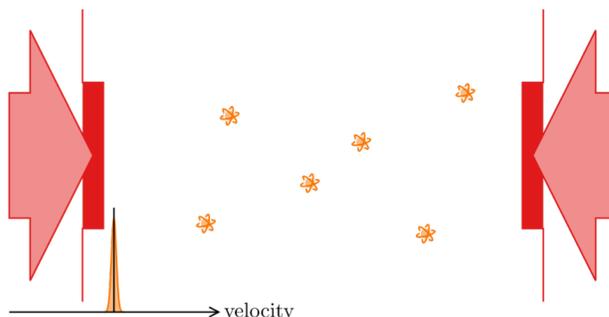


**Figure 1:** Interference fringes before **(a)** and after **(b)** velocity selection.



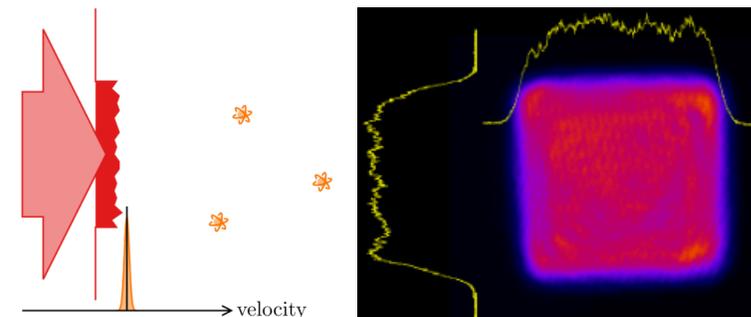
## Problem:

Range of **Rabi frequencies** from intensity variation



## Fix:

**Larger beams** require more power; “**top hat**” beams are an alternative



## But:

Still intensity fluctuations at the **~10% level**  
Can **compromise wavefront flatness**

- Inhomogeneities result in different **Rabi frequencies**, **superposition phases** and **transfer efficiencies**.

## 2. Optimal control theory

- Can we find robust pulses that maintain interferometer contrast and sensitivity in presence of inhomogeneities?
- Solution: adapt **Nuclear Magnetic Resonance optimal control theory** to design robust interferometer pulses
- Find “best” way to drive system from initial to target state(s), maximising a chosen fidelity (e.g. state-transfer)

### Define control problem

Examine Raman Hamiltonian for atom-light interaction and **identify control parameters**

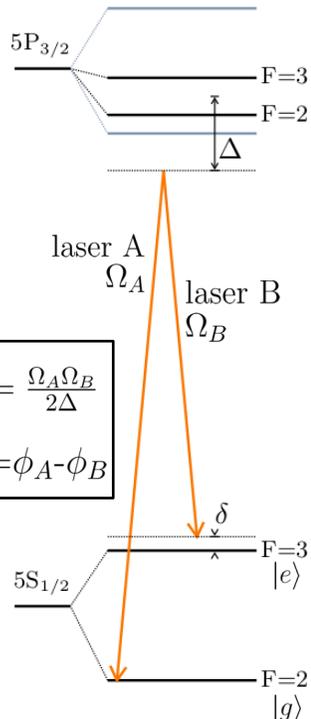
$$\hat{H}(t) = \frac{\hbar}{2} \begin{pmatrix} \delta & \Omega_R(t)e^{i\phi_L(t)} \\ \Omega_R(t)e^{-i\phi_L(t)} & -\delta \end{pmatrix}$$

Identify relative Raman laser phase as control parameter - **define piecewise constant waveform**

$$\phi_L(t) = \phi_1, \phi_2, \dots, \phi_N$$

Optimise pulse waveform to drive initial state to final state, **maximising “fidelity” [1]** :

$$\mathcal{F} = f(\langle \psi_{target} | \underbrace{\hat{U}_N \hat{U}_{N-1} \dots \hat{U}_1}_{\text{Pulse Propagator}} | \psi_{initial} \rangle)$$



$$\Omega_R = \frac{\Omega_A \Omega_B}{2\Delta}$$

$$\phi_L = \phi_A - \phi_B$$

(above) Raman transitions in <sup>85</sup>Rb

### Algorithm:

1. Guess pulse, fix duration and number of steps:

$$\phi_{\text{guess}} = \{\phi_1^i, \phi_2^i, \dots, \phi_N^i\}$$

2. Choose fidelity:

$$\mathcal{F} = |\langle e | \hat{U}_{\phi_N} \hat{U}_{\phi_{N-1}} \dots \hat{U}_{\phi_1} | g \rangle|^2$$

3. Replace with ensemble average for **robustness**:

$$\mathcal{F} \rightarrow \sum_{\delta, \Omega_R} \mathcal{F}(\delta, \Omega_R)$$

4. Compute derivatives of fidelity w.r.t. controls [2]:

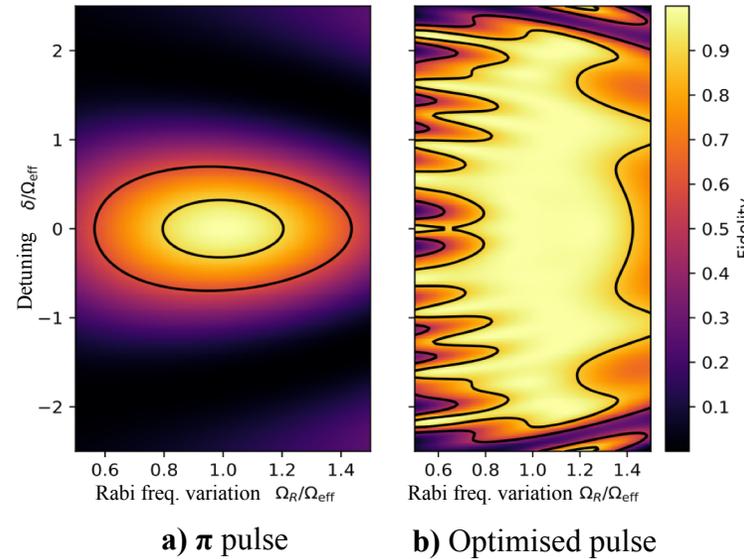
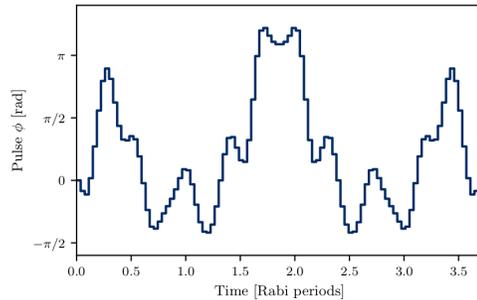
$$\partial \mathcal{F} / \partial \phi_i$$

5. Use **gradient ascent** or **quasi-Newton method** [3] to iteratively update pulse profile until target fidelity (and robustness) is reached

# 3. Optimised Raman pulses - Implementation

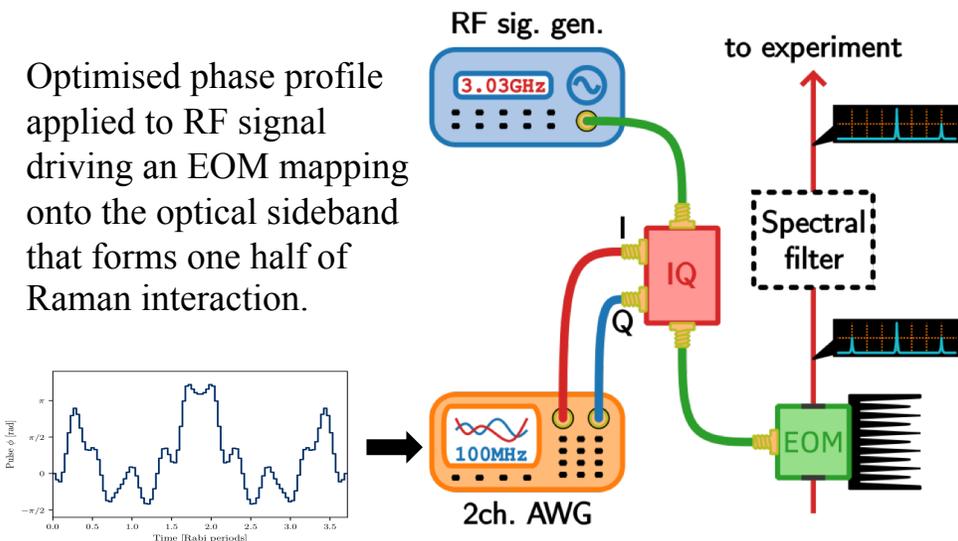
- Optimise a state-transfer pulse with fidelity  $\mathcal{F} = |\langle e | \hat{U} | g \rangle|^2$
- Magneto-optical Trap used to create  $120 \mu\text{K}$  sample of  $^{85}\text{Rb}$  distributed over Zeeman sublevels = inhomogeneous sample

**Figure 2:** Robust state-transfer pulse *phase profile* found using optimal control. Find symmetry in solutions.

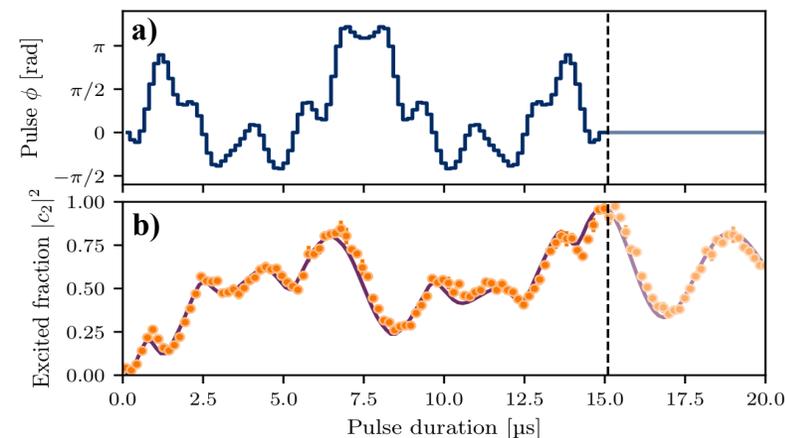


**Figure 3:** Simulated state-transfer fidelity for (a)  $\pi$  pulse and (b) optimised pulse from Fig. 2 as a function of **detuning** and **Rabi freq. variation**. The optimised pulse is highly robust to both classes of error.

Optimised phase profile applied to RF signal driving an EOM mapping onto the optical sideband that forms one half of Raman interaction.



Prepare atoms in ground state and measure **excited fraction** of atoms at **different times** throughout the pulse.



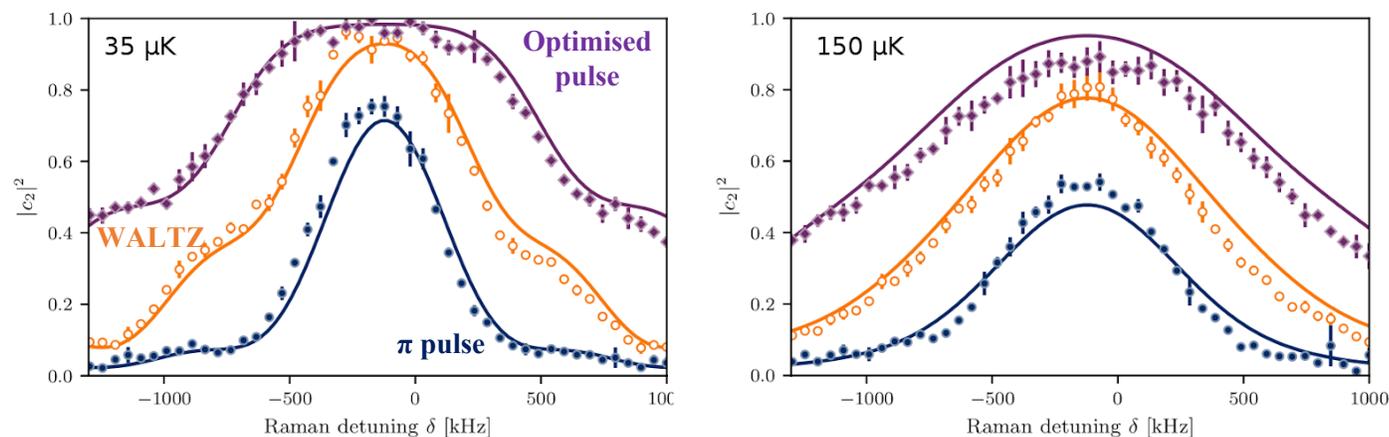
**Figure 4:** (a) Pulse waveform and (b) excited state population at different times throughout the pulse. Observe excellent agreement with model (purple line).

# 4. Optimised Raman pulses - Results

## Verifying Robustness

- Sub-Doppler sample: optimised pulse fidelity **99.8(3)%** on resonance [4] (c.f. **96(2)%** for WALTZ pulse, **75(3)%** for a  $\pi$  pulse)
- **>90% transfer over ~400kHz** range of detunings (~100kHz for WALTZ)
- Increased velocity acceptance ideal for **LMT applications** [5, 6]

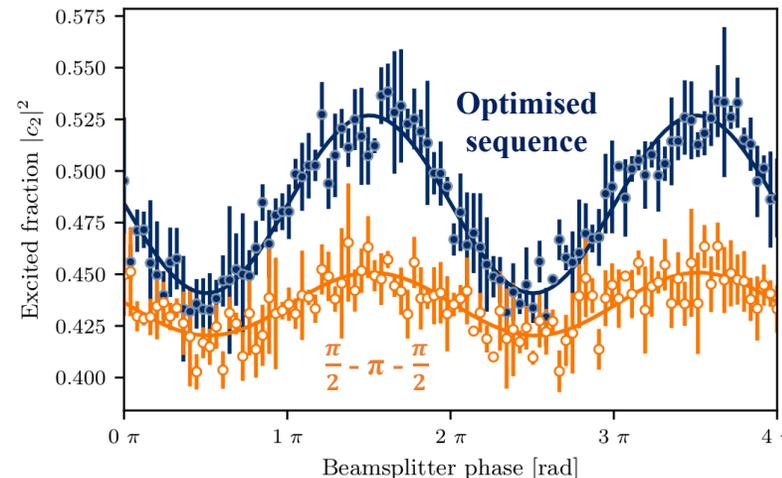
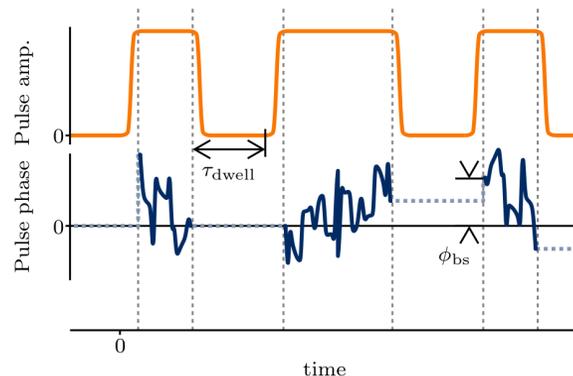
**Figure 5:** Fraction of atoms transferred to excited state by  $\pi$  pulse (blue), composite WALTZ (orange) and optimised GRAPE pulse (purple) as a function of laser detuning at two different temperatures.



## Increasing contrast

- Optimise Mach-Zehnder sequence for high contrast fringes in a warm atom cloud [4]
- Use **pulse symmetry** to cancel errors between pulses
- Observe **threefold contrast enhancement** in a ~100  $\mu\text{K}$  atom cloud compared with  $\frac{\pi}{2} - \pi - \frac{\pi}{2}$  sequence

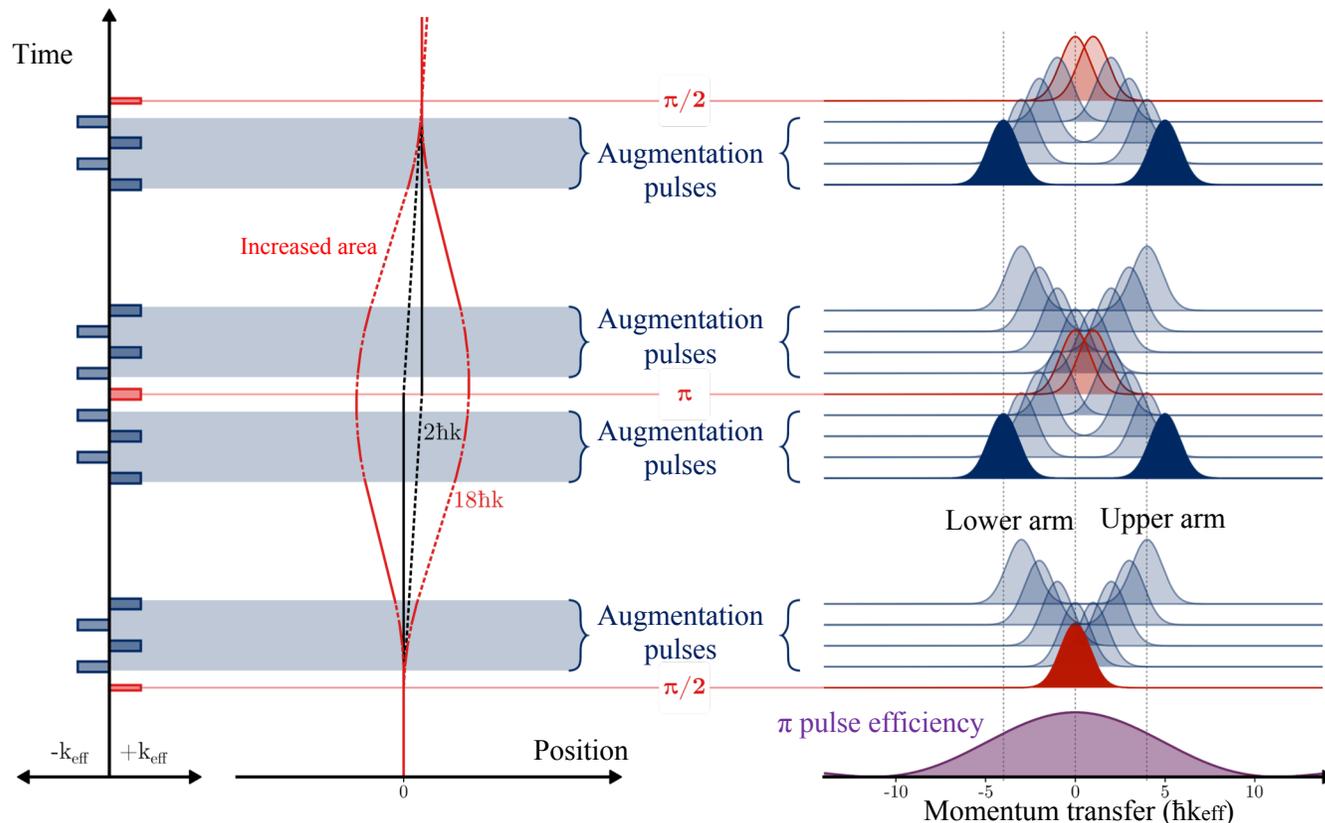
**Figure 6:** Experimental timings and pulse waveforms for optimised Mach-Zehnder sequence.



**Figure 7:** Interference fringes in a ~100  $\mu\text{K}$  atom cloud with rectangular (orange) and optimised pulses (blue). Find **3x improvement** in contrast.

# 5. Large Momentum Transfer (LMT)

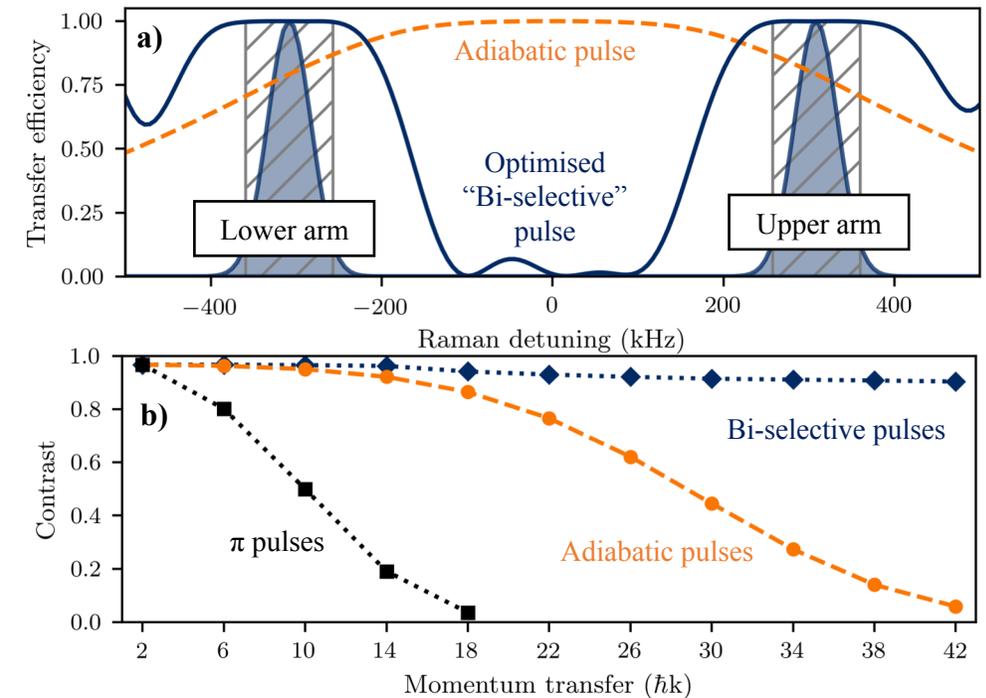
- Interferometer sensitivity can be increased by extending the pulse sequence with **augmentation pulses** forming a **large momentum transfer (LMT)** interferometer
- LMT interferometers have arms with very different momenta meaning pulses need large **velocity acceptance**



**Figure 8:** Atomic state trajectories within a LMT atom interferometer (left) and the resulting variation in atomic momentum distribution throughout the sequence (right).

**Problem:** Each successive pulse must provide efficient state-transfer for arms which are increasingly separated in detuning

**Solution:** Individually optimise each augmentation pulse [7] to provide efficient transfer for evolving momentum distribution. Result is significant increase in contrast within LMT sequences → **more momentum can be transferred to atoms before contrast lost**



**Figure 9:** (a) State-transfer efficiency of bi-selective pulse (blue) and adiabatic pulse (orange) against detuning. (b) Simulated LMT contrast for different sequences as momentum separation of wavepackets is increased.