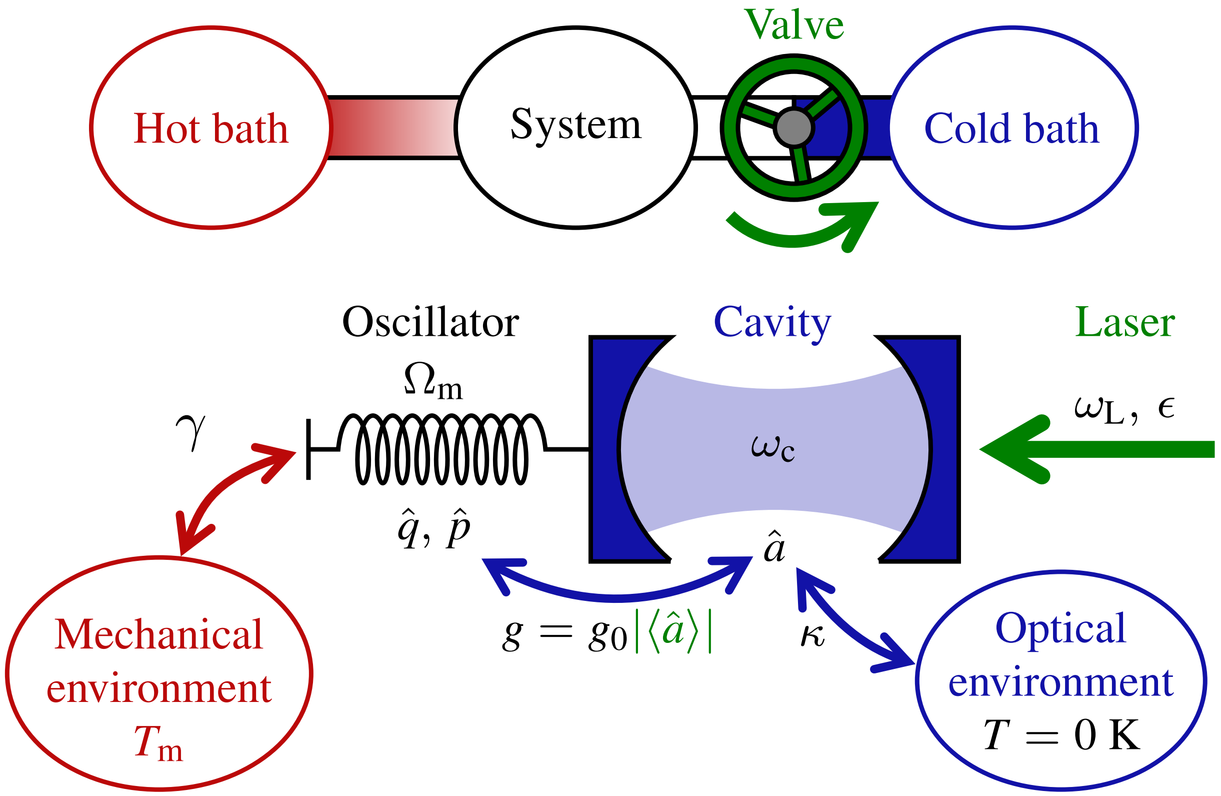
Optomechanical cooling efficiency: the cost of turning a valve

Juliette Monsel[a*], Nastaran Dashti[a], Sushanth Kini Manjeshwar[a], Janine Splettstoesser[a] and Witlef Wieczorek[a]
[a] Department of Microtechnology and Nanoscience (MC2), Chalmers University of Technology, S-412 96 Göteborg, Sweden
* monsel@chalmers.se



Platform: Cavity optomechanics

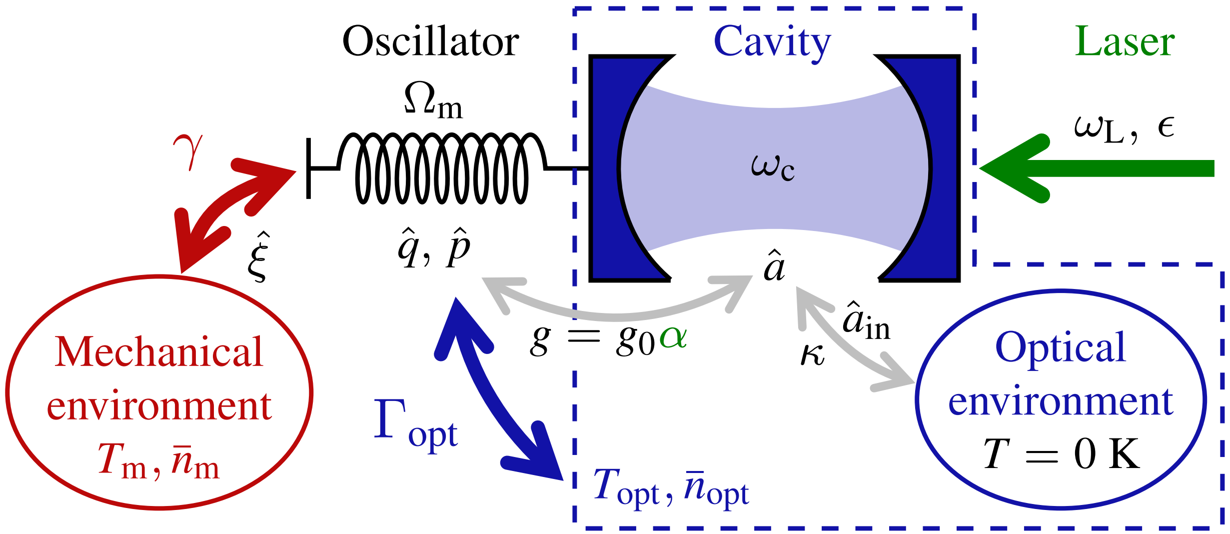
Protocol: Cooling down the mechanical oscillator in the resolved-sideband regime ($\kappa \ll \Omega_m$)

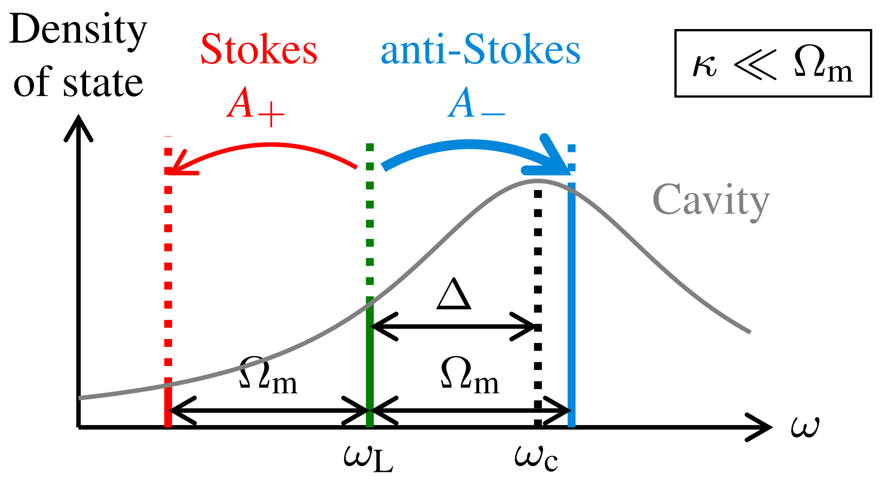
- How can we understand resolved-sideband cooling from a thermodynamic perspective?

Improved cooling schemes including using squeezed light or a Fano mirror have been proposed:

- How do they compare?
- Are they more efficient than the original scheme?

1. Resolved-sideband cooling





$$H = \hbar\Delta\hat{a}^\dagger\hat{a} + \frac{\hbar\Omega_m}{2}(\hat{q}^2 + \hat{p}^2) - \hbar g_0\sqrt{2}\hat{q}\hat{a}^\dagger\hat{a} + \hbar(\epsilon\hat{a}^\dagger + \epsilon^*\hat{a})$$

Linearization[1]:

$\hat{a} = \alpha + \delta\hat{a}$

$\hat{q} = \bar{q} + \delta\hat{q}$

$\hat{p} = \bar{p} + \delta\hat{p}$

$\alpha = -i\epsilon/(\kappa + i\tilde{\Delta})$

$\bar{q} = \sqrt{2}g_0|\alpha|^2/\Omega_m$

$\bar{p} = 0$

$\tilde{\Delta} = \omega_c - \omega_L - \sqrt{2}g_0\bar{q}$

$$H = \hbar\tilde{\Delta}\delta\hat{a}^\dagger\delta\hat{a} + \frac{\hbar\Omega_m}{2}(\delta\hat{q}^2 + \delta\hat{p}^2) - \hbar g\sqrt{2}\delta\hat{q}(\delta\hat{a} + \delta\hat{a}^\dagger)$$

Scattering processes:

- Stokes: photon ω_L $\rightarrow$ phonon Ω_m + photon $\omega_L - \Omega_m$
- anti-Stokes: photon ω_L + phonon Ω_m $\rightarrow$ photon $\omega_L + \Omega_m$

Dynamics:

Langevin equations[1]

$$\delta\dot{\hat{q}} = \Omega_m\delta\hat{p}$$

$$\delta\dot{\hat{p}} = -\Omega_m\delta\hat{q} - \gamma\delta\hat{p} + g\sqrt{2}(\delta\hat{a}^\dagger + \delta\hat{a}) + \sqrt{\gamma}\hat{\xi}$$

$$\delta\dot{\hat{a}} = -(\kappa + i\tilde{\Delta})\delta\hat{a} + ig\sqrt{2}\delta\hat{q} + \sqrt{2\kappa}\hat{a}_{in}$$

Noises

$$\langle\hat{\xi}(t)\hat{\xi}(t')\rangle = (2\bar{n} + 1)\delta(t - t')$$

$$\langle\hat{a}_{in}(t)\hat{a}_{in}^\dagger(t')\rangle = \delta(t - t')$$

1. C. Genes et al. *Phys. Rev. A* **77**, 033804 (2008)

2. Thermodynamic analysis

Weak coupling regime ($g \ll \kappa, \Omega_\mathrm{m}$):

Steady state phonon number:

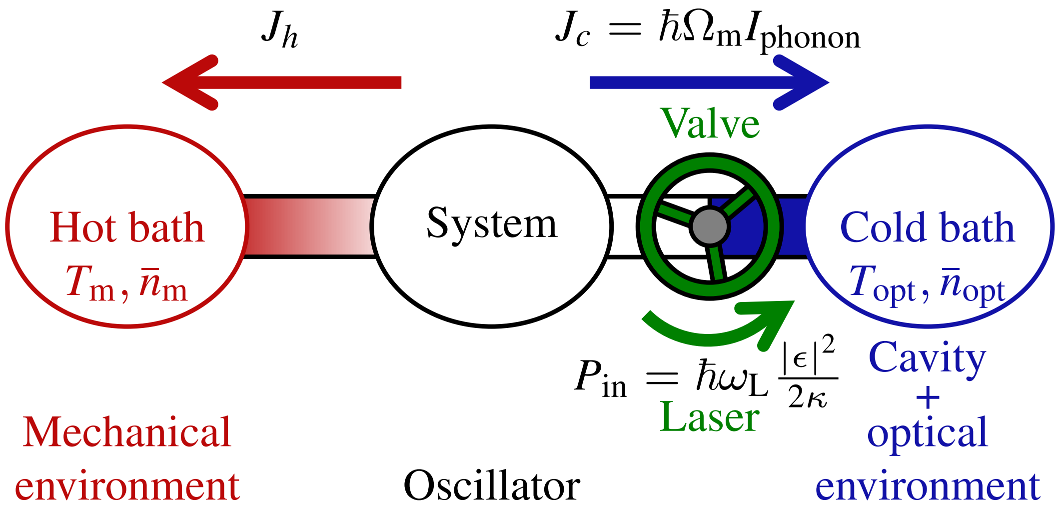
$$n_\mathrm{eff} = \langle \delta\hat{b}^\dagger \delta\hat{b} \rangle_\mathrm{ss} = \frac{\gamma \bar{n}_\mathrm{m} + \Gamma_\mathrm{opt}\bar{n}_\mathrm{opt}}{\gamma + \Gamma_\mathrm{opt}}$$

with $\Gamma_\mathrm{opt} = A_- - A_+$, $\bar{n}_\mathrm{opt} = A_+/\Gamma_\mathrm{opt}$

$\Rightarrow$ Optical part behaves like a cold phonon bath at effective temperature T_opt such that $\bar{n}_\mathrm{opt} = (e^{\hbar\Omega_\mathrm{m}/k_B T_\mathrm{opt}} - 1)^{-1}$

1. I. Wilson-Rae et al. *New J. Phys.* **10**, 095007 (2008)

Thermodynamic roles:



Beyond the weak coupling regime:

$$\frac{\mathrm{d}\langle \delta\hat{a}^\dagger \delta\hat{a}\rangle}{\mathrm{d}t} = \underbrace{ig\left(\langle \delta\hat{a}^\dagger(\delta\hat{b}^\dagger + \delta\hat{b})\rangle - \langle \delta\hat{a}(\delta\hat{b}^\dagger + \delta\hat{b})\rangle\right)}_{I_\mathrm{photon}} - 2\kappa\langle \delta\hat{a}^\dagger \delta\hat{a}\rangle$$

$$\frac{\mathrm{d}\langle \delta\hat{b}^\dagger \delta\hat{b}\rangle}{\mathrm{d}t} = \underbrace{ig\left(\langle \delta\hat{b}^\dagger(\delta\hat{a} + \delta\hat{a}^\dagger)\rangle - \langle \delta\hat{b}(\delta\hat{a}^\dagger + \delta\hat{a})\rangle\right)}_{I_\mathrm{phonon}} + \gamma(\bar{n} - \langle \delta\hat{b}^\dagger \delta\hat{b}\rangle) + \frac{\gamma}{2}\left(1 + \langle \delta\hat{b}^2\rangle + \langle (\delta\hat{b}^\dagger)^2\rangle\right)$$

... and the equations for all the other 2nd order moments $\Rightarrow$ steady-state

Efficiencies:

- Resource = flux of photons interacting with the mechanics

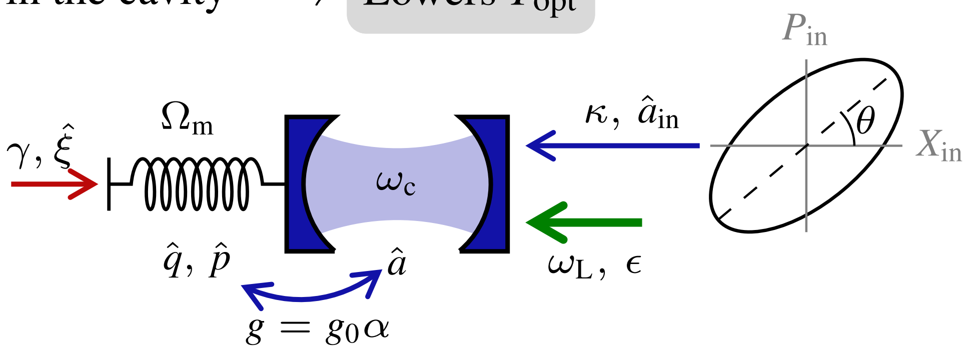
$$\eta_\mathrm{conv} = \frac{\hbar\Omega_\mathrm{m} I_\mathrm{phonon}}{\hbar\tilde{\Delta} I_\mathrm{photon}}$$

- Resource = laser power

$$\eta_\mathrm{laser} = \frac{\hbar\Omega_\mathrm{m} I_\mathrm{phonon}}{P_\mathrm{in}}$$

3. Improved cooling schemes

Squeezed light: Squeeze the input light to decrease the amplitude noise in the cavity[1,2] $\Rightarrow$ Lowers T_{opt}



Cavity's input noise:

$$\langle \hat{a}^{\dagger}_{\mathrm{in}}(t)\hat{a}_{\mathrm{in}}(t')\rangle = N\delta(t-t')$$
$$\langle \hat{a}_{\mathrm{in}}(t)\hat{a}_{\mathrm{in}}(t')\rangle = M\delta(t-t')$$

with

$$N = \frac{4r_s^2}{(1-r_s^2)^2}, \quad M = \frac{2r_s(1+r_s^2)}{(1-r_s^2)^2}\mathrm{e}^{-i2\theta},$$

and squeezing ratio $0 < r_s < 1$.

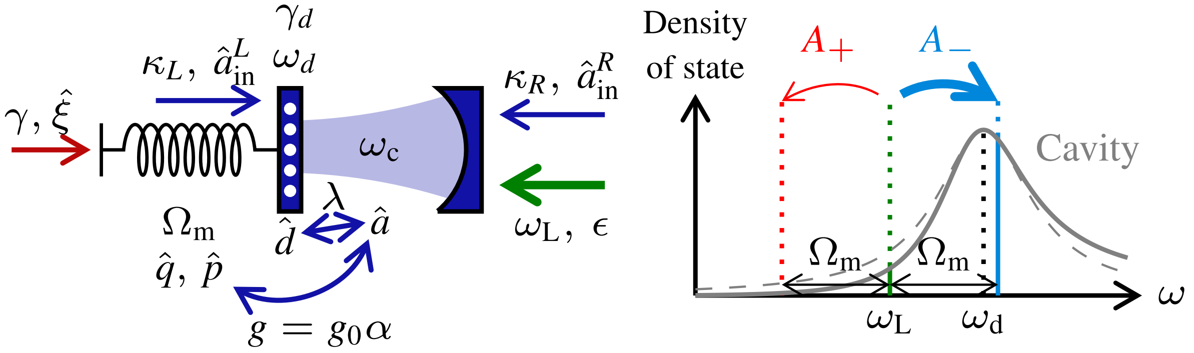
$\Rightarrow$ Optimal θ and r_s minimizes amplitude noise

$\Rightarrow$ **Cost:** Squeezing generation

1. M. Asjad et al. *Phys. Rev. A* **94**, 051801 (2016)

2. J. B. Clark et al. *Nature* **541**, 191 (2017)

Fano mirror: Asymmetrize the cavity's lineshape to suppress the Stokes process[3] $\Rightarrow$ Lowers T_{opt} and increases Γ_{opt}



$$\delta\dot{\hat{q}} = \Omega_{\mathrm{m}}\delta\hat{p},$$
$$\delta\dot{\hat{p}} = -\Omega_{\mathrm{m}}\delta\hat{q} - \gamma\delta\hat{p} + g\sqrt{2}(\delta\hat{a}^{\dagger} + \delta\hat{a}) + \sqrt{\gamma}\hat{\xi}$$
$$\delta\dot{\hat{a}} = -(\kappa + i\tilde{\Delta})\delta\hat{a} + ig\sqrt{2}\delta\hat{q} - \mathcal{G}\delta\hat{d} + \sqrt{2\kappa_L}\hat{a}^L_{\mathrm{in}} + \sqrt{2\kappa_R}\hat{a}^R_{\mathrm{in}}$$
$$\delta\dot{\hat{d}} = -(\gamma_d + i\tilde{\Delta}_d)\delta\hat{d} - \mathcal{G}\delta\hat{a} + \sqrt{2\gamma_d}\hat{a}^L_{\mathrm{in}}$$

with $\mathcal{G} = i\lambda + \sqrt{\kappa_L\gamma_d}$, $\kappa = \kappa_L + \kappa_R$

$\Rightarrow$ Optimal γ_d simultaneously maximize cooling and minimizing heating for $\tilde{\Delta}_d \simeq \Omega_{\mathrm{m}}$

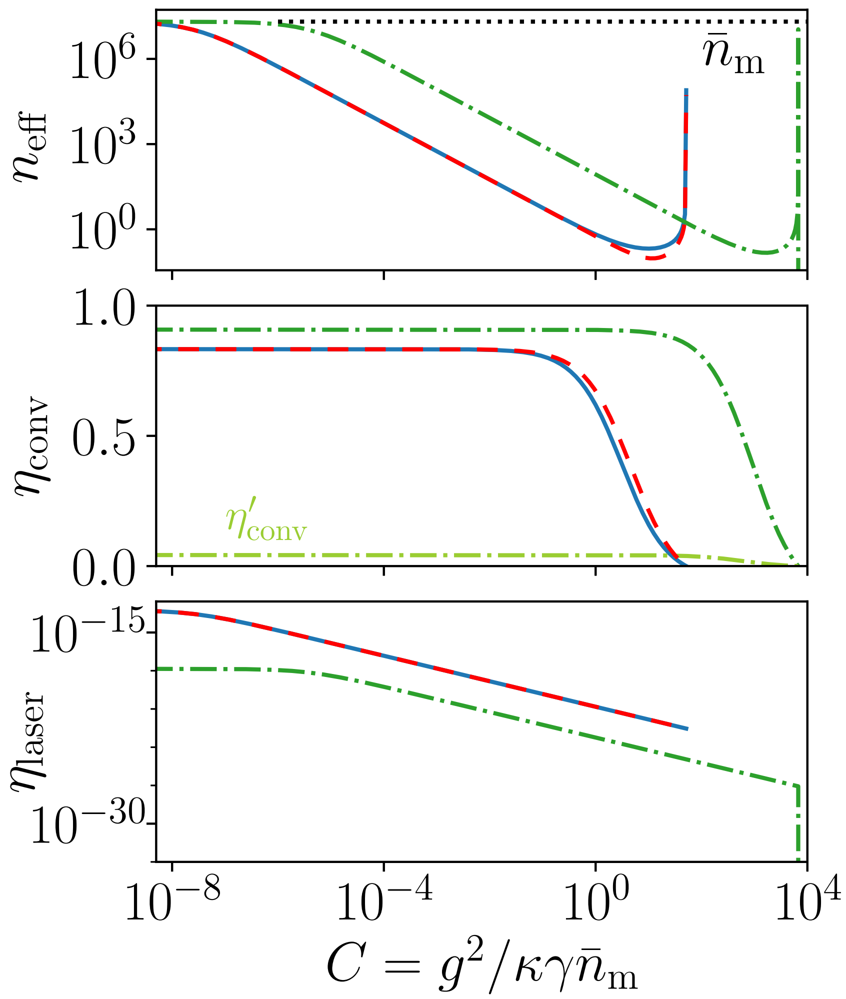
$\Rightarrow$ **Cost:** Build field in Fano mirror's mode

$$\eta'_{\mathrm{conv}} = \frac{\Omega_{\mathrm{m}} g\,\mathrm{Im}(\langle\delta\hat{b}^{\dagger}(\delta\hat{a}^{\dagger}+\delta\hat{a})\rangle)}{\tilde{\Delta}\left(g\,\mathrm{Im}(\langle\delta\hat{a}(\delta\hat{b}^{\dagger}+\delta\hat{b})\rangle) - \mathrm{Re}(\mathcal{G}\langle\delta\hat{d}\delta\hat{a}^{\dagger}\rangle)\right)}$$

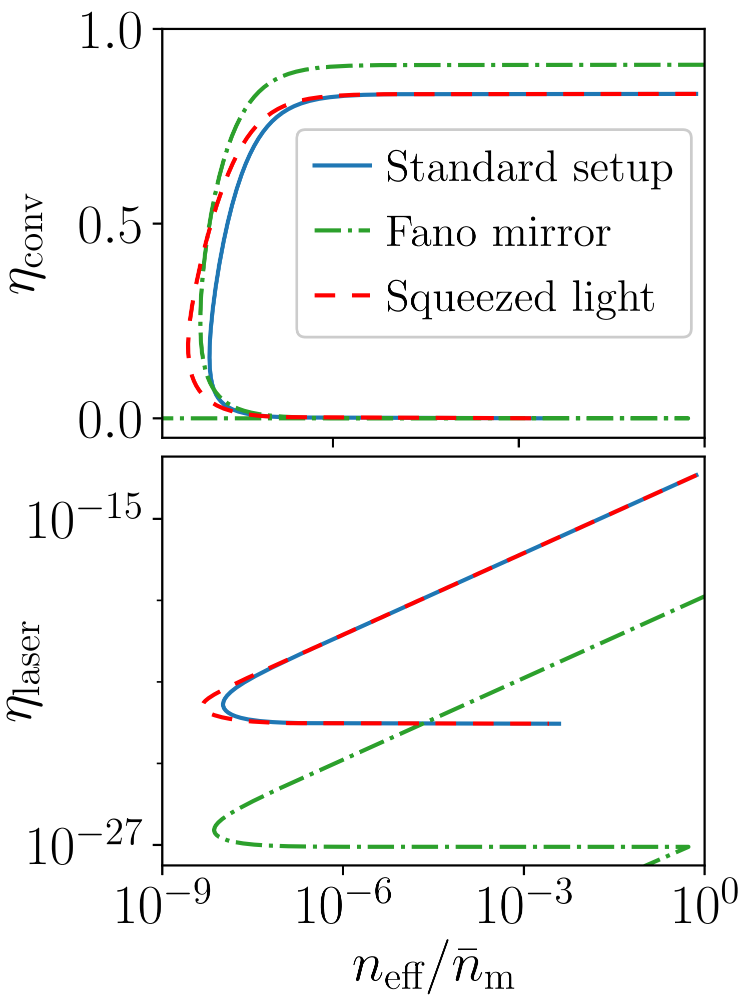
3. O. Černotík et al. *Phys. Rev. Lett.* **122**, 243601 (2019)

4. Phonon number and efficiencies

As functions of cooperativity C



Efficiency versus phonon number



$$\eta_{\mathrm{conv}} = \frac{\hbar\Omega_{\mathrm{m}} I_{\mathrm{phonon}}}{\hbar\tilde{\Delta} I_{\mathrm{photon}}} \text{ and } \eta_{\mathrm{laser}} = \frac{\hbar\Omega_{\mathrm{m}} I_{\mathrm{phonon}}}{\hbar\omega_{\mathrm{L}}|\epsilon|^2/2\kappa}$$

Parameters: (Levitated optomechanics)

$$\begin{aligned} \Omega_{\mathrm{m}}/2\pi &= 305\ \mathrm{kHz} & \lambda_{\mathrm{L}} &= 1064\ \mathrm{nm} \\ T_{\mathrm{m}} &= 300\ \mathrm{K} & \tilde{\Delta} &= \Omega_{\mathrm{m}} \\ \gamma/2\pi &= 1.6\cdot 10^{-4}\ \mathrm{Hz} & \kappa/2\pi &= 193\ \mathrm{kHz} \\ \bar{n}_{\mathrm{m}} &= 1.3\cdot 10^{8} & \kappa/\Omega_{\mathrm{m}} &= 0.63 \\ Q_{\mathrm{m}} &= 1.9\cdot 10^{9} & g_0/2\pi &= 0.3\ \mathrm{kHz} \\ C_0 &= 2.2\cdot 10^{-11} & & \end{aligned}$$

U. Delić et al. *Science* **367**, 892 (2020)

Squeezing: $r_s = 0.16$, $\theta = 0.89$ rad (optimal)

Fano mirror: $\tilde{\Delta}_d = \Omega_{\mathrm{m}}$, $\gamma_d/2\pi = 3.86$ MHz (optimal), other parameters chosen to have an effective cavity linewidth = κ

Squeezed light: Smallest n_{eff}

- $\Rightarrow$ Efficiencies barely impacted
- $\Rightarrow$ Cost of squeezing generation not taken into account

Fano mirror: Smaller n_{eff} (but $> n_{\mathrm{eff}}^{\mathrm{squeezing}}$)

- $\Rightarrow$ Significant decrease in efficiency
- $\Rightarrow$ Cost of building a coherent field in the Fano mirror's mode

5. Conclusion

- Cavity + Optical environment = Effective cold phonon bath
- Optomechanical coupling $g = g_0\alpha$ is like a valve

  $\Rightarrow$ Cost to couple the cold bath to the system: Build coherent field α in the cavity
- Combining dynamics, beyond the weak coupling regime, with thermodynamic analysis

  $\Rightarrow$ Efficiencies η_{conv} (conversion photon $\rightarrow$ phonon) and η_{laser} (resource = laser power)