Entanglement measure from the Fubini-Study metric for arbitrary M-qudit hybrid systems

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Outline

The new entanglement measure (EM) proposed here applies to

- M qubit
- M qudit
- M hybrid

pure and mixed state.

Properties

The proposed entanglement measure

- is invariant under local unitary transformations.
- has an explicit computable expression.
- In the case of M-qubit systems, it has the physical interpretation of an obstacle to the minimum distance between infinitesimally close states.
- In the case of M-qubit systems, the analysis of the eigenvalues of the metric tensor associated with the entanglement measure allows one to quantify the robustness of the entanglement of a state.

Definition

The EM is based on a geometric approach.

The d-dimensional Hilbert space of an hybrid M qudit system is equipped with the Fubini-Study (FS) metric [3]

$$\langle d\psi|d\psi\rangle - \frac{1}{4}|\langle\psi|d\psi\rangle - \langle d\psi|\psi\rangle|^2$$

for $|\psi\rangle$ normalized and $|d\psi\rangle$ infinitesimal variation of such state.

Since the distance must not be affected by local unitary operations U on a given state $|s\rangle$, the FS metric reads

$$\sum_{\mu\nu} g_{\mu\nu}(\mathbf{v})d\xi^\mu d\xi^\nu = \sum_{\mu\nu}\big(\langle s|(\mathbf{v}\cdot\mathbf{T})_\mu(\mathbf{v}\cdot\mathbf{T})_\nu|s\rangle + -\langle s|(\mathbf{v}\cdot\mathbf{T})_\mu|s\rangle\langle s|(\mathbf{v}\cdot\mathbf{T})_\nu|s\rangle\big)\, d\xi^\mu d\xi^\nu$$

Where $\mathbf{v}_\nu\cdot\mathbf{T}_\nu = U_\nu^\dagger \mathbf{n}_\nu\cdot\mathbf{T}_\nu U_\nu$.

$\mathbf{n}_\mu$ is and unit vector in $\mathbb{R}^{d_\mu}$ and $T_{\mu a}$ are the generators of $\mathbf{su}(d_\mu)$ algebra.

Entanglement measure for pure states [1]

M qubit states

$$E(|s\rangle) = \inf_{\{\mathbf{v}_\mu\}_\mu} \mathrm{tr}\,(g(\mathbf{v}))$$

M hybrid qudit states

$$E(|s\rangle) = \sum_{\mu=0}^{M-1}\left[\frac{2(d_\mu-1)}{d_\mu} - \sum_{k=1}^{d_\mu^2-1}\langle s|T_{\mu k}|s\rangle^2\right]$$

Entanglement measure for mixed states

For a mixed state $\rho = \sum_j p_j|\psi_j\rangle\langle\psi_j|$,

where p_j is the probability of measuring the state $|\psi_j\rangle$, the measure is [1]

$$E(\rho) = min\sum_j p_j E(|\psi_j\rangle)$$

where the minimum is taken over all the possible representation ρ.

Examples of applications

Briegel Raussendorf M-qubit states [2]

$$|r,\phi\rangle_M = U(\phi)\,|r,0\rangle$$

where the entanglement operator

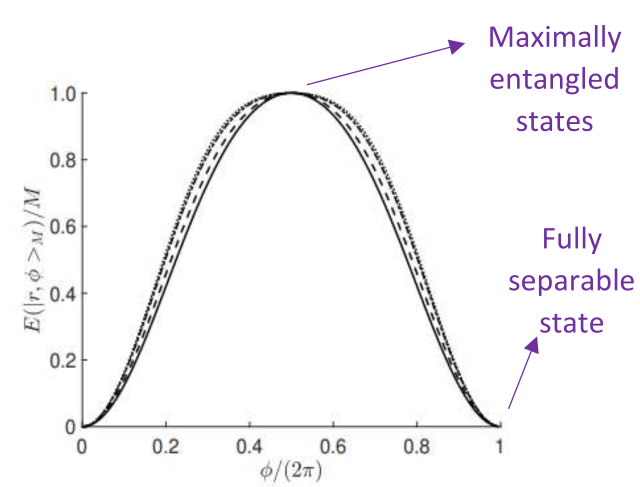
$$U(\phi) = \exp\left(-i\phi\sum_{j=1}^{M-1}\Pi_0^j\Pi_1^{j+1}\right)$$

generates the entangled states $|r,\phi\rangle_M$ starting from

$$|r,0\rangle = \bigotimes_{j=0}^{M-1}\frac{1}{\sqrt{2}}\left(|0\rangle_j + |1\rangle_j\right)$$

with $\Pi_0^j = (\mathbb{1}+\sigma_{j3}/2)$ and $\Pi_1^j = (\mathbb{1}-\sigma_{j3}/2)$.

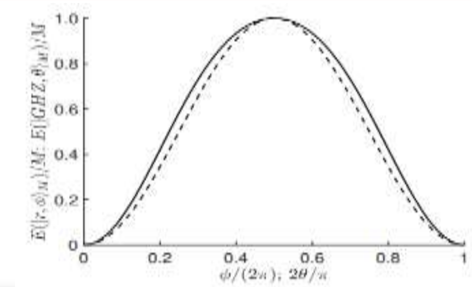
- The state is fully separable for $\phi = 2\pi k,\ k\in\mathbb{Z}$
- The state is maximally entangled for $\phi = (2k+1)\pi$



GHZ M-qubit states

$$|GHZ,\theta\rangle_M = \cos(\theta)|0\rangle + \sin(\theta)e^{i\varphi}|2^M-1\rangle$$

- The state is fully separable for $\theta = k\pi/2$ and $\forall\varphi$
- The state is maximally entangled for $\theta = k\pi/2 + \pi/4$ $(\forall\varphi)$



Concluding remarks

The proposed new entanglement measure has successfully passed a testing procedure, in which

- the inequivalence of some maximally entangled states, already demonstrated in the literature, was retrieved.
- a comparison with Von Neumann's entropy has been done and the advantages have been assessed.

Acknowledgment

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[1] Cocchiarella, Scali, Ribisi, Nardi, Bel-Hadj-Aissa, and Franzosi, **Entanglement distance for arbitrary M-qudit hybrid systems,** Physical Review A 101, 042129 (2020).

[2] Briegel and Raussendorf, **Persistent entanglement in arrays of interacting particles,** Phys. Rev. Lett. 86, 910 (2001).

[3] Gibbons, **Typical states and density matrices**, Journal of Geometry and Physics 8, 147 (1992).