Symmetry analysis and multipole classification of eigenmodes in electromagnetic resonators

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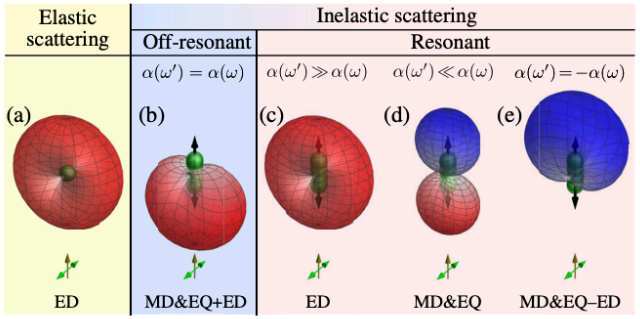
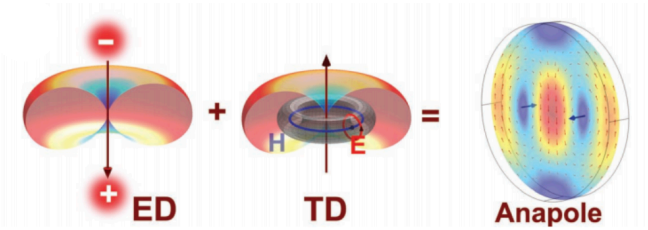
department of PHYSICS

Main idea | Method 1 (based on RSE) | Method 2 (based on Wigner theorem) | Results | Conclusion

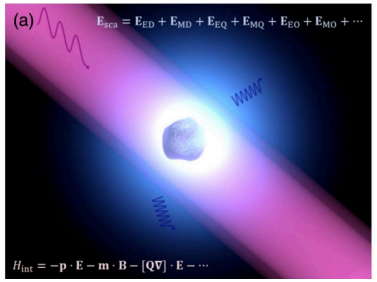
Applications of the multipolar approach



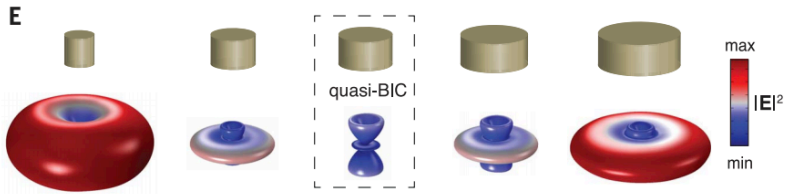
Phys. Rev. X 9, 011008 (2019)



Advanced Optical Materials 2019, 7, 1801350

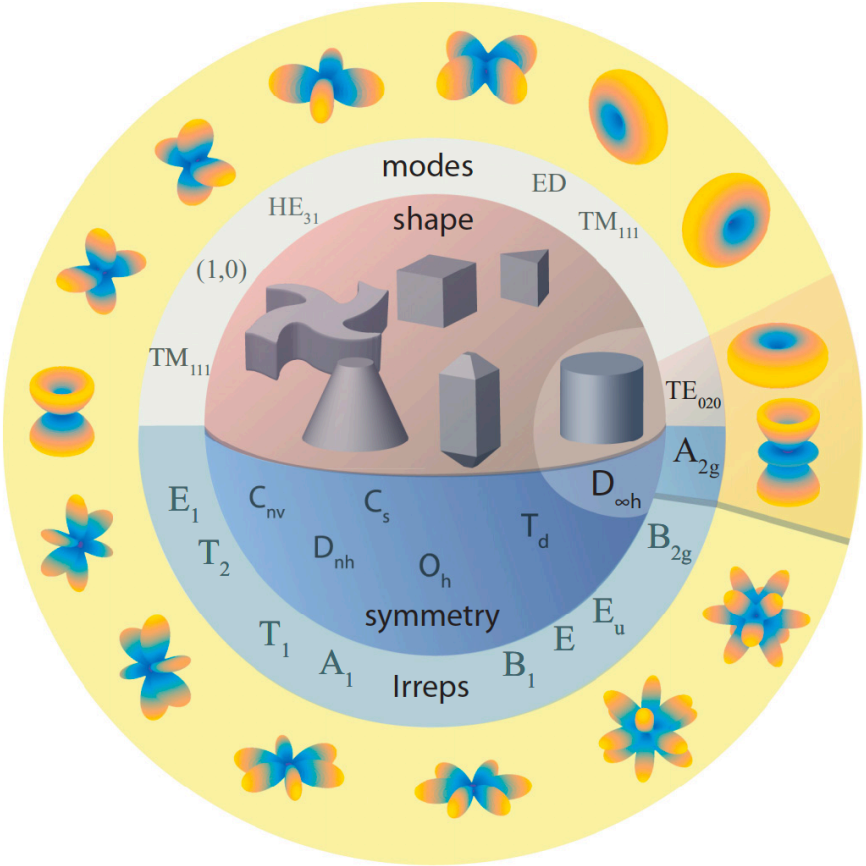


Optica Vol. 3, Issue 11, pp. 1241-1255 (2016)



Science 17 Jan 2020: Vol. 367, Issue 6475, pp. 288-292

What is the multipolar content of resonator's eigenmodes ?



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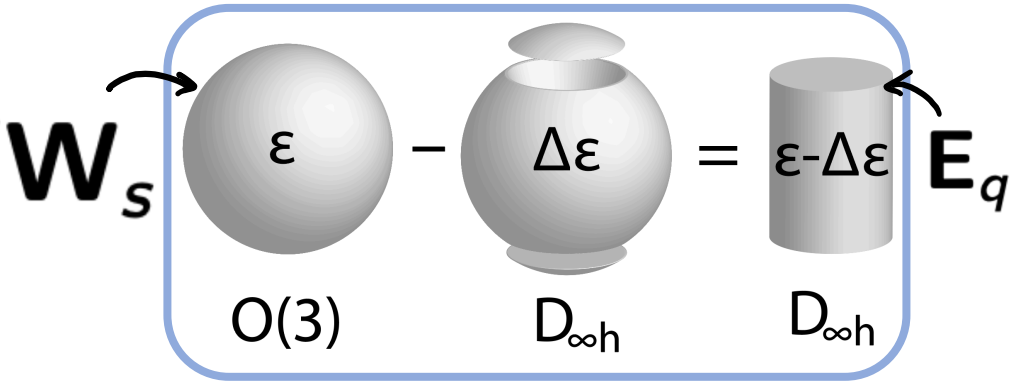
Resonant state expansion (RSE)



$$\nabla \times \nabla \times \mathbf{E}_q = [\varepsilon(\mathbf{r}) - \Delta\varepsilon(\mathbf{r})]\frac{\Omega_q^2}{c^2}\mathbf{E}_q$$

Resonator's mode $\mathbf{E}_q$: sum of the resonant states of sphere $\mathbf{W}_s$

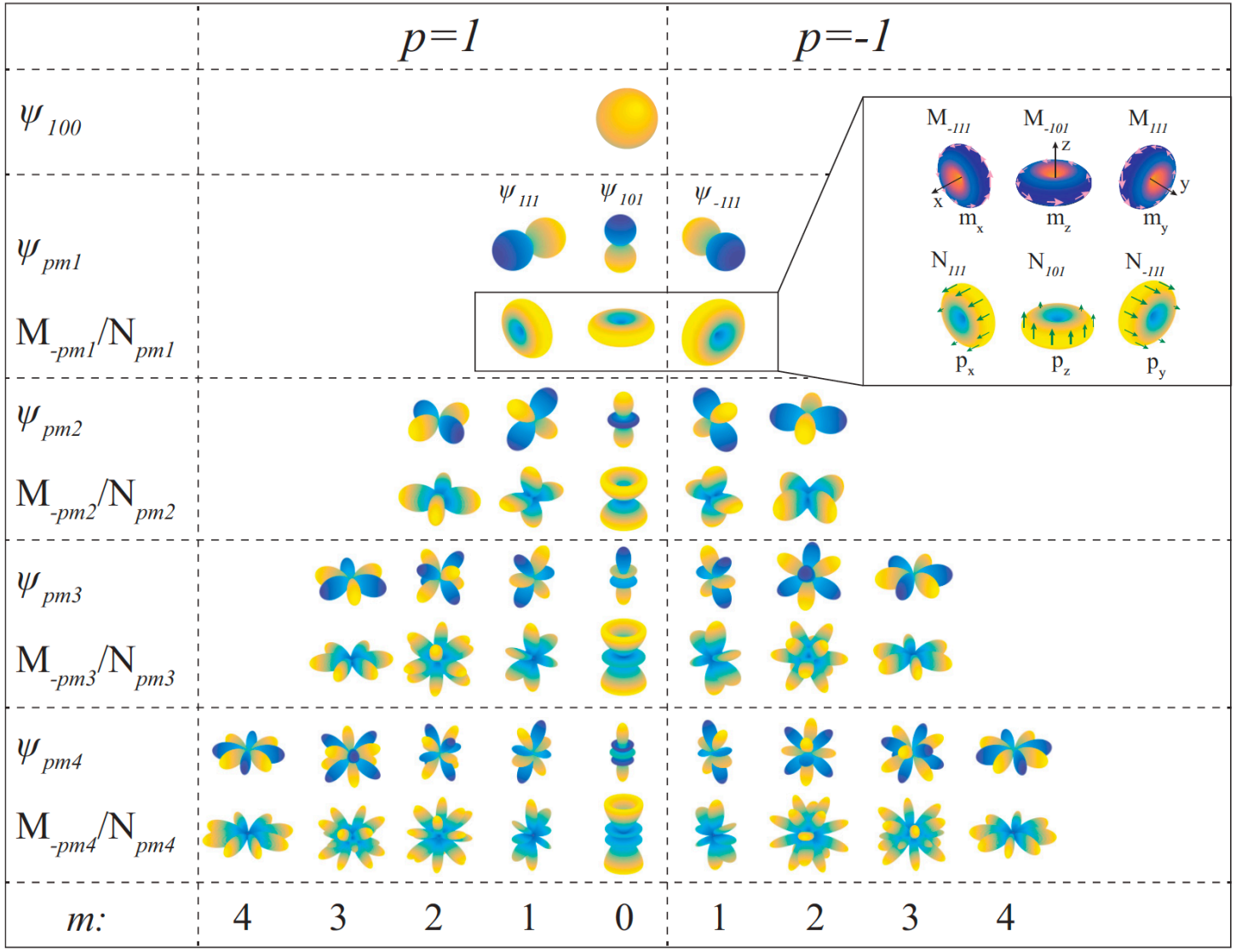
$$\mathbf{E}_q = \sum_s C_s^q \mathbf{W}_s$$

Perturbation theory:

$$V_{ss'} \sim \int \Delta\varepsilon(\mathbf{r})\mathbf{W}_s(\mathbf{r}) \cdot \mathbf{W}_{s'}(\mathbf{r})d\mathbf{r}$$

M. B. Doost, W. Langbein, and E. A. Muljarov
Phys. Rev. A 90, 013834 (2014)

$W_s = N_{pmn}$, M_{pmn} - vector spherical harmonics



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"Addition of angular momenta"

$$\boxed{V_{ss'} \sim \int \Delta\varepsilon(\mathbf{r})\mathbf{W}_s(\mathbf{r}) \cdot \mathbf{W}_{s'}(\mathbf{r})d\mathbf{r} \quad s - p, m, n}$$

$$\mathbf{N}_{pmn} \cdot \mathbf{N}_{p'm'n'} = \sum a_{p''n''m''}(r)\psi_{p''n''m''}$$

$$p'' = pp'$$

$$m'' = |m \pm m'|$$

$$|n - n'| \leqslant n'' \leqslant n + n'$$

If at least one function $\psi_{p''m''n''}$ is an **invariant**, i.e. transforms into itself under all transformations of the symmetry group, then **the integral** $V_{ss'}$ **is not equal to zero.**

$$V_{ss'} \sim \int \Delta\varepsilon(\mathbf{r})\mathbf{W}_s(\mathbf{r}) \cdot \mathbf{W}_{s'}(\mathbf{r})d\mathbf{r} \neq 0$$

And if the integral is not equal to zero, **both multipoles belong to one mode**

Algorithm

1. Find the functions $\psi_{p''m''n''}$, which are invariant with respect to symmetry transformations of the resonator
2. Take an arbitrary function $\mathbf{W}_{pmn}$
3. Find harmonics $\mathbf{W}_{p'm'n'}$ coupled to $\mathbf{W}_{pmn}$ using the relations:

$$p'' = pp'$$

$$m'' = |m \pm m'|$$

$$|n - n'| \leqslant n'' \leqslant n + n'$$

+conservation of the inversion parity

4. Profit! The multipoles $\mathbf{W}_{p'm'n'}$ and $\mathbf{W}_{pmn}$ belong to one mode

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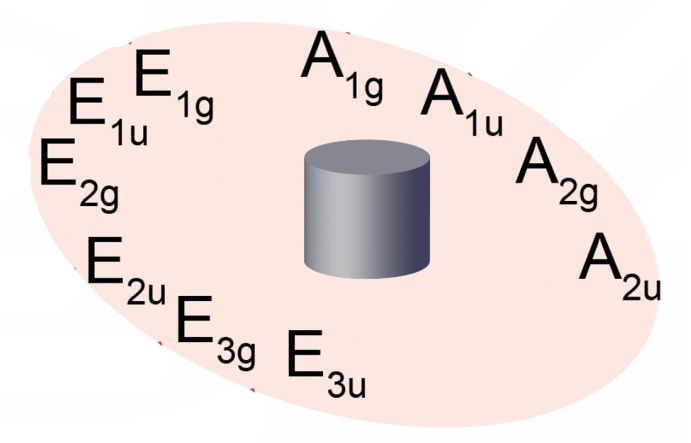
Wigner theorem

Wigner theorem

The eigenmodes are transformed by irreducible representations (irreps) of group symmetry of a particle

(each mode corresponds to one irrep and can be named by the notation of the irrep)



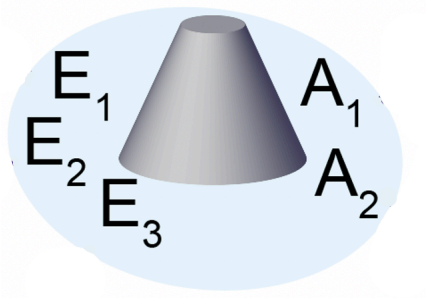


Table of the irreps

Algorithm

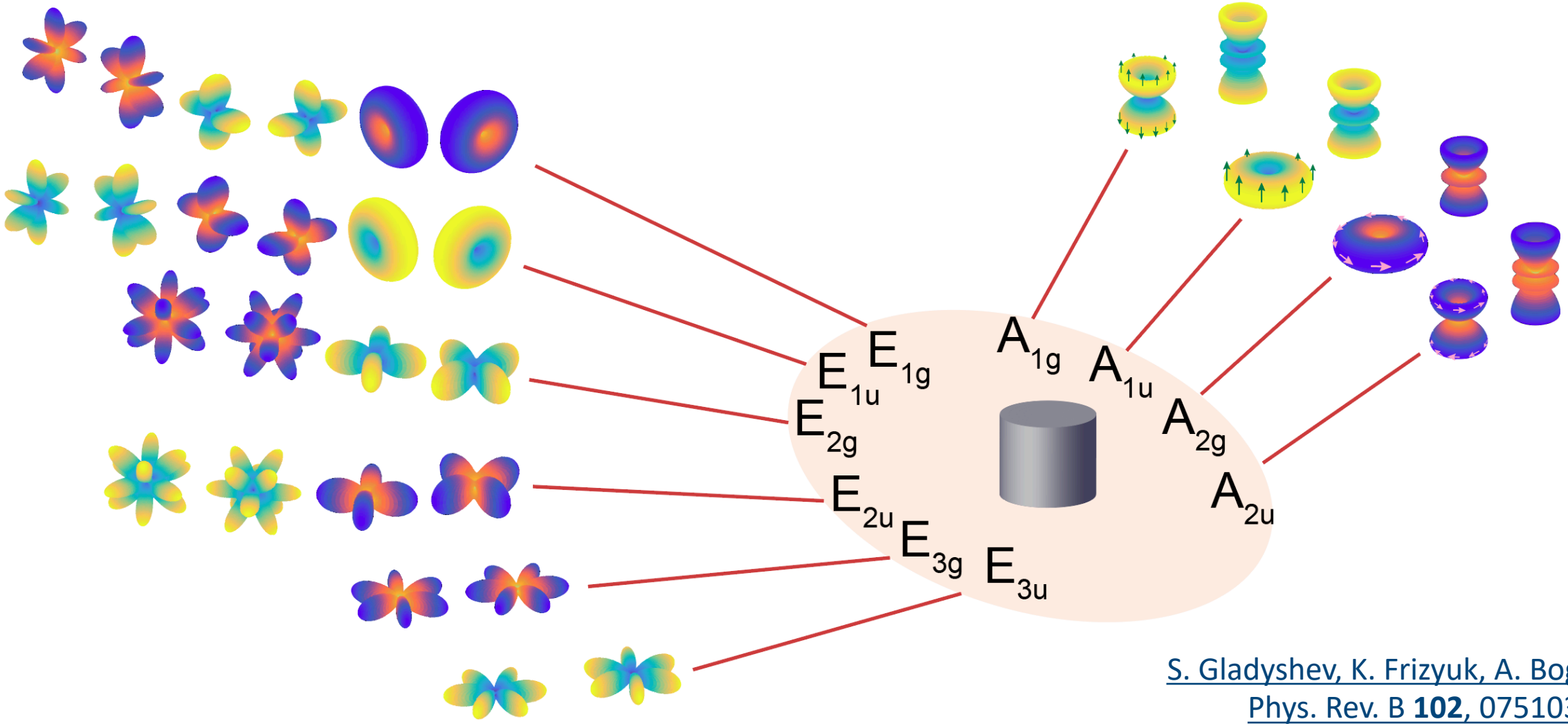
Just find all vector spherical harmonics, which are transformed by particular irreducible representation of the resonator's symmetry group, and they will all belong to one mode

Relations between scalar multipoles and irreps can be found, for example, here:
http://gernot-katzers-spice-pages.com/character_tables/

For vector spherical harmonics, inversion symmetry must be taken into account.

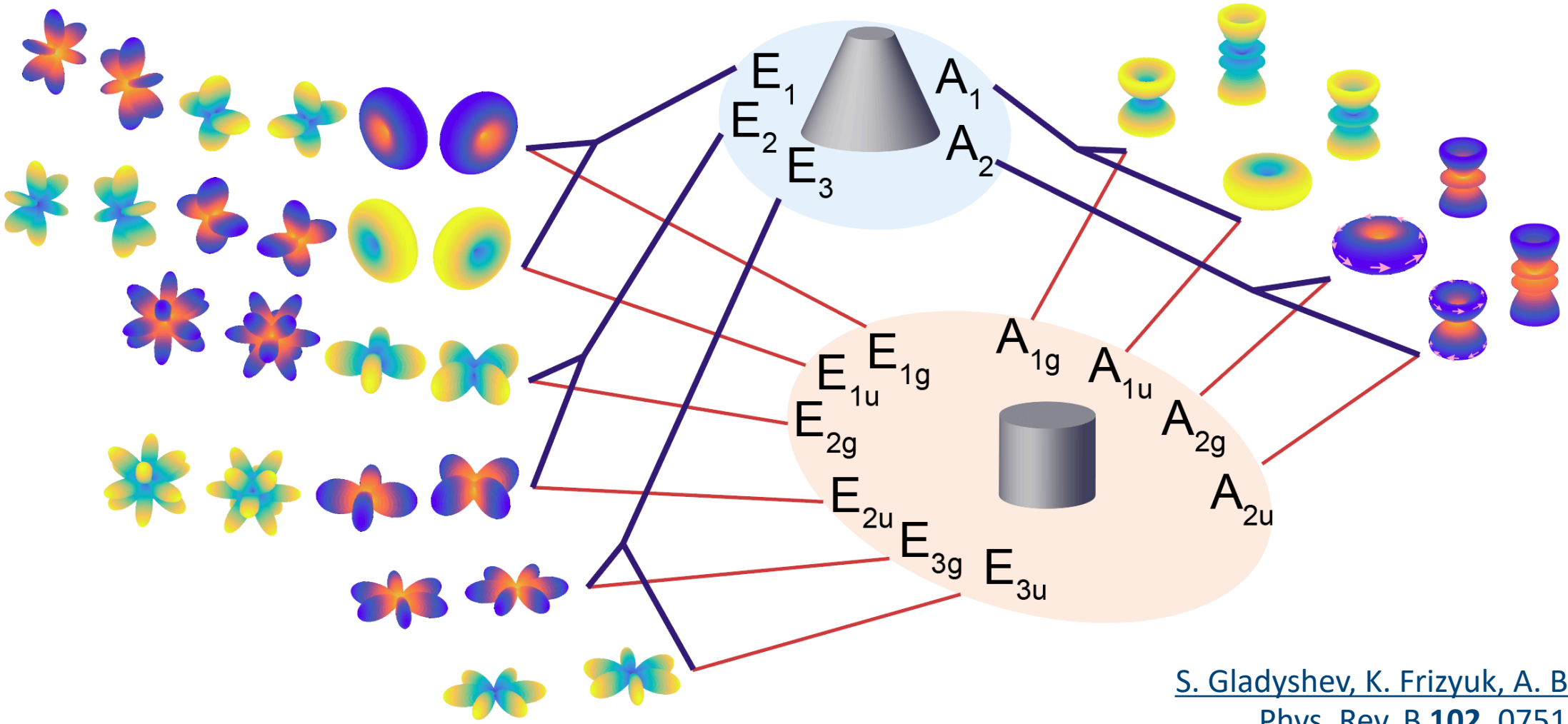
Optics Express Vol. 28, Issue 3, pp. 3073-3085 (2020)

Relation between modes and multipoles for a cylinder



S. Gladyshev, K. Frizyuk, A. Bogdanov
Phys. Rev. B **102**, 075103, 2020

Relation between modes and multipoles for a cylinder and a cone



S. Gladyshev, K. Frizyuk, A. Bogdanov
Phys. Rev. B **102**, 075103, 2020

1. Each mode correspond to particular irreducible representation and consist of particular infinite set of multipoles

2. Set of multipoles for each mode can be found by two methods, provided in our work

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Telegram: @kuyzirf

Our paper: S. Gladyshev, K. Frizyuk, A. Bogdanov Phys. Rev. B **102**, 075103, 2020
https://arxiv.org/abs/2002.11411

$\psi_{emn}(kr) \propto P_n^m(\cos\theta)\cos m\varphi$

$\psi_{omn}(kr) \propto P_n^m(\cos\theta)\sin m\varphi$