Symmetry analysis and multipole classification of eigenmodes in electromagnetic resonators

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Main idea
Method 1 (based on RSE)  Method 2 (based on Wigner theorem)  Results
Conclusion

Applications of the multipolar approach

What is the multipolar content of resonator’s eigenmodes?

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Method 1 (based on RSE)

Resonant state expansion (RSE)

\[ W_s = \sum C_s^q W_s \]

Resonator’s mode \( E_q \): sum of the resonant states of sphere \( W_s \)

Perturbation theory:

\[ V_{ss'} \sim \int \Delta \varepsilon(r) W_s(r) \cdot W_{s'}(r) dr \]

M. B. Doost, W. Langbein, and E. A. Muljarov

Method 2 (based on Wigner theorem)

Results

Conclusion

\[ W_s = N_{pmn}, M_{pmn} - \text{vector spherical harmonics} \]

\( p=1 \)

\( p=-1 \)
“Addition of angular momenta”

\[ V_{ss'} \sim \int \Delta \varepsilon(r) \mathbf{W}_s(r) \cdot \mathbf{W}_{s'}(r) \, dr \quad s - p, m, n \]

If at least one function \( \Psi_{p''m''n''} \) is an invariant, i.e. transforms into itself under all transformations of the symmetry group, then the integral \( V_{ss'} \) is not equal to zero.

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And if the integral is not equal to zero, both multipoles belong to one mode.

Algorithm

1. Find the functions \( \Psi_{p''m''n''} \), which are invariant with respect to symmetry transformations of the resonator.
2. Take an arbitrary function \( \mathbf{W}_{pmn} \).
3. Find harmonics \( \mathbf{W}_{p'm'n'} \) coupled to \( \mathbf{W}_{pmn} \) using the relations:
   \[ p'' = pp' \]
   \[ m'' = |m \pm m'| \]
   \[ |n - n'| \leq n'' \leq n + n' \]
   +conservation of the inversion parity
4. Profit! The multipoles \( \mathbf{W}_{p'm'n'} \) and \( \mathbf{W}_{pmn} \) belong to one mode.
Wigner theorem

The eigenmodes are transformed by irreducible representations (irreps) of group symmetry of a particle

(each mode corresponds to one irrep and can be named by the notation of the irrep)

Algorithm

Just find all vector spherical harmonics, which are transformed by particular irreducible representation of the resonator’s symmetry group, and they will all belong to one mode.

Relations between scalar multipoles and irreps can be found, for example, here:

http://gernot-katzers-spice-pages.com/character_tables/

For vector spherical harmonics, inversion symmetry must be taken into account.

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Relation between modes and multipoles for a cylinder

S. Gladyshev, K. Frizyuk, A. Bogdanov
Phys. Rev. B 102, 075103, 2020
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Relation between modes and multipoles for a cylinder and a cone
1. Each mode correspond to particular irreducible representation and consist of particular infinite set of multipoles

2. Set of multipoles for each mode can be found by two methods, provided in our work

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Our paper: S. Gladyshev, K. Frizyuk, A. Bogdanov Phys. Rev. B 102, 075103, 2020
\[ \psi_{enm}(kr) \propto P^m_n(\cos \theta) \cos m\varphi \]

\[ \psi_{omn}(kr) \propto P^m_n(\cos \theta) \sin m\varphi \]

\[ \psi_{e00} \]

\[ \psi_{pm1} \]

\[ \psi_{pm2} \]

\[ \psi_{pm3} \]

\[ \psi_{pm4} \]

\[ \psi_{pm5} \]

\[ \psi_{pm6} \]

\[ m: \ 6 \ 5 \ 4 \ 3 \ 2 \ 1 \ 0 \]

\[ 0 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \]