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Perfect absorption by a single spherical nanoparticle

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The perfect absorption occurs when the of radiative and dissipative decay rates in the system are equal.

Classical systems that exhibit a perfect absorption phenomenon require extended objects or near-field components in incident fields.

The most widely known example of a system with perfect absorption is the Salisbury Screen [1], where the phenomenon appears for the thin screens placed near the conducting plane.

Other solutions suggest using other planar systems with critical coupling [2] or involve the use of multiple coherent sources that produce convergent cylindrical or spherical waves, which necessary to have near-field components [3].

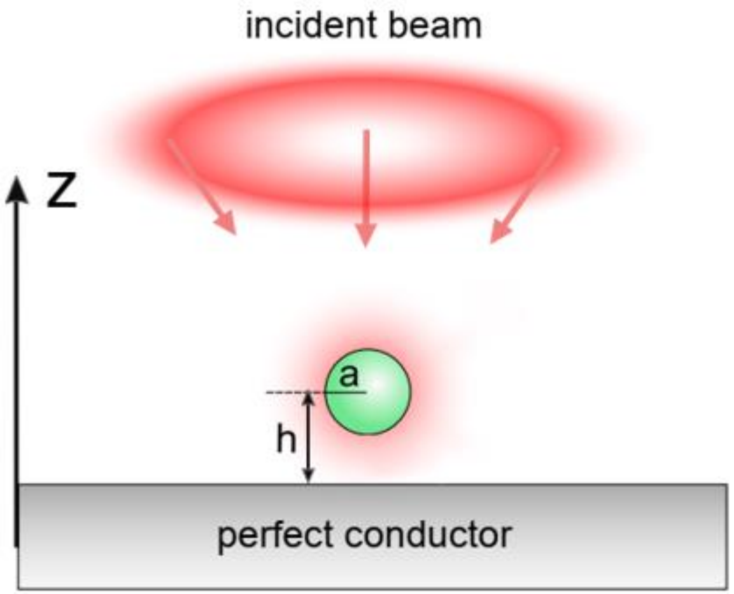
References

[1] Fante, R. L., & McCormack, M. T. (1988). *IEEE transactions on antennas and propagation*, 36(10), 1443-1454.

[2] Zhu, L., Liu, F., Lin, H., Hu, J., Yu, Z., Wang, X., & Fan, S. (2016). *Light: Science & Applications*, 5(3), e16052-e16052.

[3] L. Novotny and B. Hecht, *Principles of Nano-Optics* (Cambridge University Press, 2006).

Our solution requires a single spherical particle located above the conductive substrate and the radially polarized cylindrical illumination beam with no near-field radiation:



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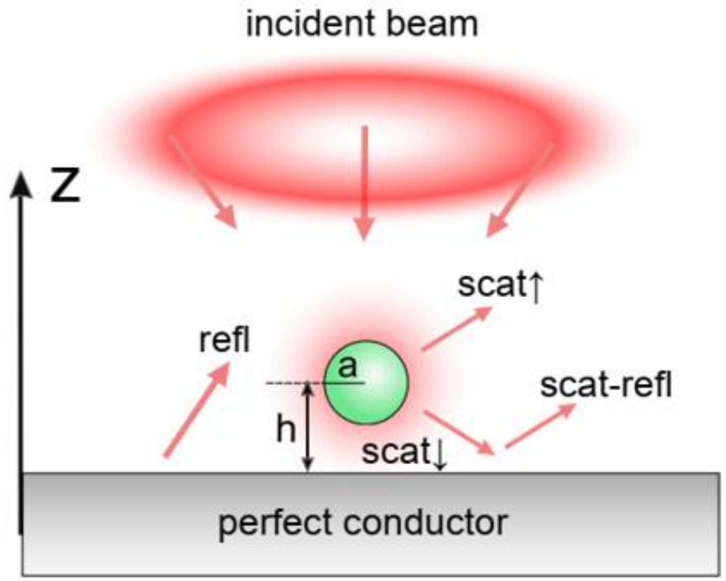
Multiple-scattering approach



$$\mathbf{E} = \mathbf{E}^{\mathrm{inc}} + \mathbf{E}^{\mathrm{refl}} + \mathbf{E}^{\mathrm{scat}} + \mathbf{E}^{\mathrm{scat-refl}}$$

References

[4] P. C. Waterman and R. Truell, J. Math. Phys. **2**, 512 (1961).

A perfect absorption condition

Because of the symmetry of the problem, the problem effectively becomes effectively scalar. The induced dipole moment relates to the total electric field at the scatterer location:

$$p_z = \varepsilon_0 \alpha_0 \left(E_z^{\mathrm{inc}} + E_z^{\mathrm{refl}} + E_z^{\mathrm{scat-refl}}\right)_{\mathbf{r=0}}$$

Despite scattered-reflected field itself depends on the p_z, one can introduce a ‘dressed’ polarizability α, that makes p_z relates to the background field only by the expression

$$p_z = \varepsilon_0 \alpha \left(E_z^{\mathrm{inc}} + E_z^{\mathrm{refl}}\right)_{\mathbf{r=0}},$$

where α does not depend on the intensity of the incident field.

After writing the incident field in the form

$$E_z^{inc} = \int_0^{k_0} \tilde{E}_0(k_\rho) J_0(k_\rho \rho) e^{-ik_z z} dk_\rho$$

(the integration boundaries are chosen to describe only far-field components), it becomes possible to write down the expression for the total scattered electric field in the upper half-space

$$E_z^{\mathrm{tot-scat}} = \int_0^\infty \left[\tilde{E}_z^{\mathrm{refl}}(k_\rho) + \tilde{E}_z^{\mathrm{scat}}(k_\rho) + \tilde{E}_z^{\mathrm{scat-refl}}(k_\rho)\right] J_0(k_\rho \rho)\, e^{ik_z z} dk_\rho .$$

In the case of PEC substrate near-field components does not transfer the energy, therefore, the perfect absorption phenomenon occurs if and only If for all $0 < k_\rho < k_0$

$$\tilde{E}_z^{\mathrm{refl}}(k_\rho) + \tilde{E}_z^{\mathrm{scat}}(k_\rho) + \tilde{E}_z^{\mathrm{scat-refl}}(k_\rho) = 0.$$

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Second-kind Fredholm equation for perfect absorption

After substituting expressions for the electric fields into the perfect absorption condition, the requirement takes the form

$$\tilde{E}_0(k_\rho)R(k_\rho)e^{i\delta} + \frac{i}{4\pi\varepsilon_0}p_z\frac{k_\rho^3}{k_z}\left(1 + R(k_\rho)e^{i\delta}\right) = 0,$$

where $R(k_\rho)$ is reflection coefficient for z component of the electric field, and $\delta = 2k_z h$.

Since p_z internally depends on the value of the background field at the point of the dipole, it is the integral equation for the unknown $\tilde{E}_0(k_\rho)$. Luckily, it is the second-type Fredholm equation:

$$\tilde{E}_0(k_\rho) + \frac{i}{4\pi}\alpha \int_0^{k_0} \hat{K}(k_\rho, k'_\rho)\tilde{E}_0(k'_\rho)dk'_\rho = 0$$

with separatable kernel

$$\hat{K}(k_\rho, k'_\rho) = \frac{k_\rho^3}{k_z}\frac{1+R(k_\rho)e^{i\delta(k_\rho)}}{R(k_\rho)e^{i\delta(k_\rho)}}\left(1 + R(k'_\rho)e^{i\delta(k'_\rho)}\right).$$

By the Fredholm alternative [5], it has non-zero solutions only for a single value of α, and type of the kernel of the integral equation allows to obtain expressions for α and $\tilde{E}_0$ in a closed form:

$$\frac{1}{\hat{\alpha}} = -\frac{i}{4\pi}\int_0^{k_0}\frac{k_\rho^3}{k_z}\frac{\left(1+Re^{i\delta}\right)^2}{Re^{i\delta}}dk_\rho, \text{ and } \tilde{E}_0(k_\rho) = A\frac{k_\rho^3}{k_z}\frac{\left(1+Re^{i\delta}\right)}{Re^{i\delta}}\theta(k_0 - k_\rho),$$

where $\theta(x)$ is the Heaviside step-function.

References

[5] H. Hochstadt, *Integral Equations* (John Wiley & Sons, 2011).

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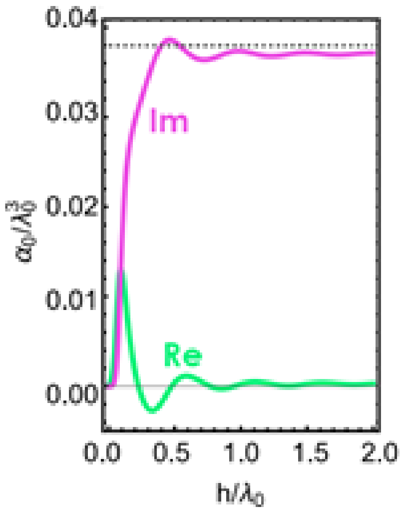
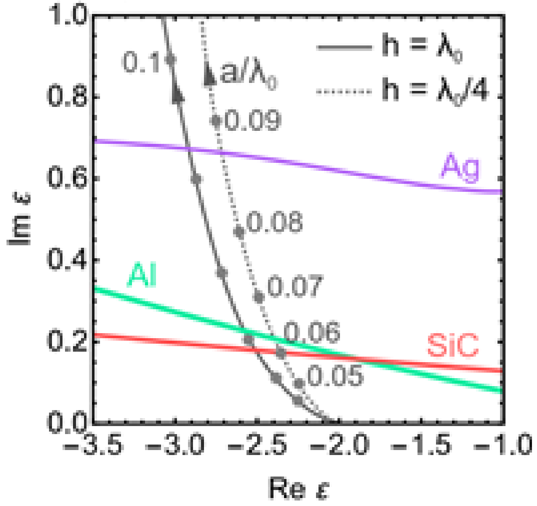
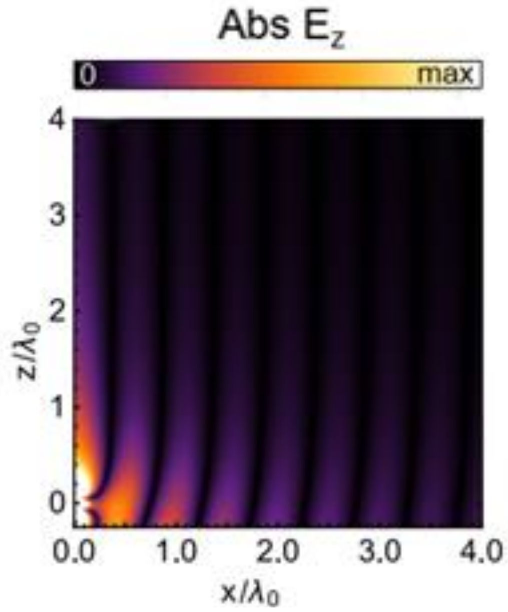
Point dipole calculations for PEC substrate



Bare electric dipole polarizability α_0 required for perfect absorption as a function of the sphere-to-substrate distance h; dashed line – polarizability of a critically coupled dipolar scatterer



Trajectories of complex permittivities yielding the perfect absorption condition parametrized with a/λ_0 for a series of values of h/λ_0. Thick lines denote complex permittivities of silver, aluminum, and silicon carbide, crossing the analytical solution in specific points.



Scattered field distribution for the $h/\lambda_0 = 1/4$. The particle is located at $\rho = z = 0$, the substrate is at $z = -h$.

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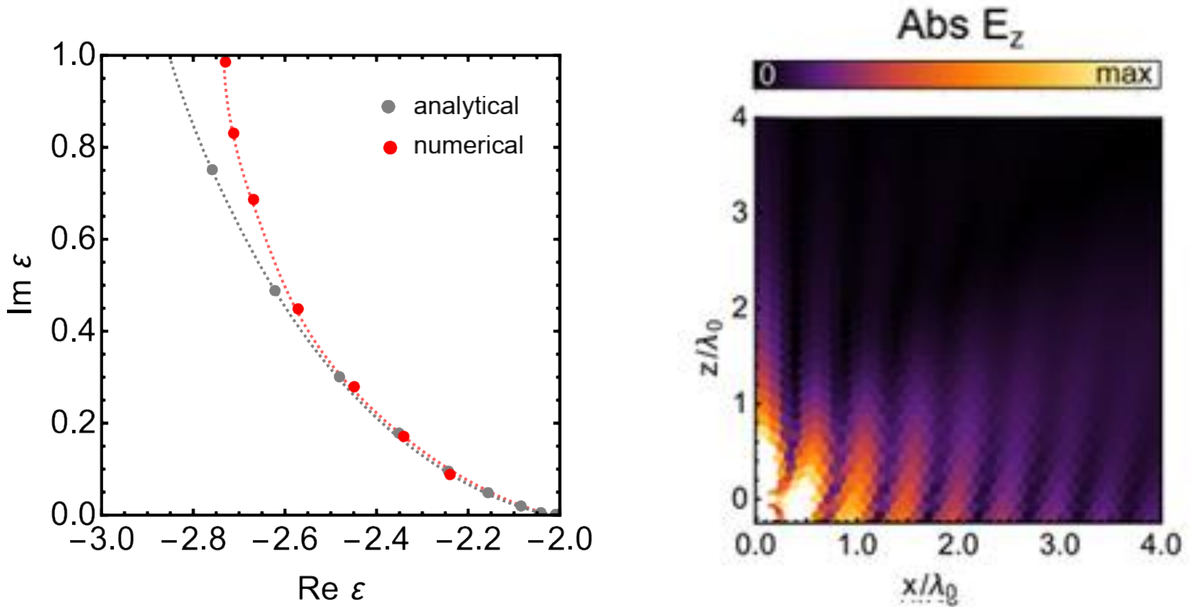
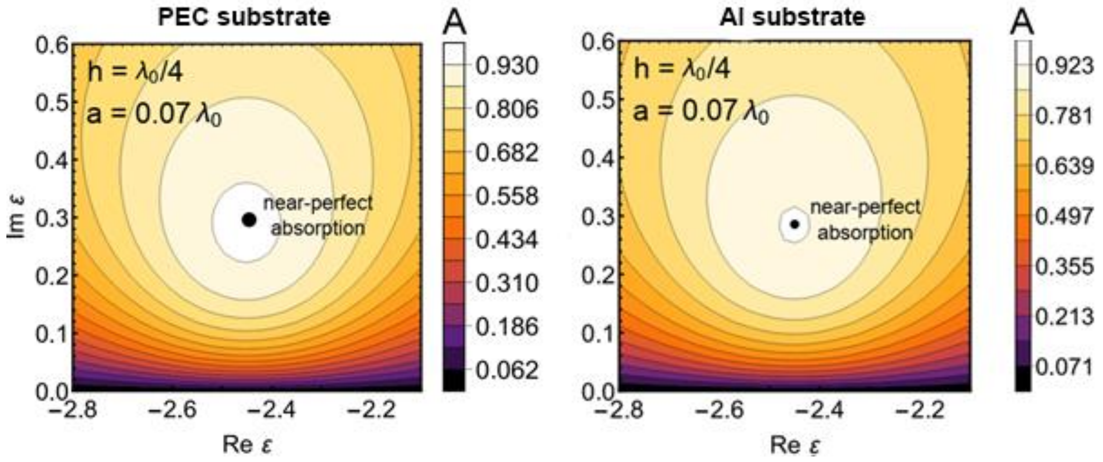
Full wave FEM simulations

We verified the obtained analytical result for the point dipolar scatterer with full-wave FEM simulations.



Left: comparison of the complex permittivities supporting the perfect absorption obtained analytically (grey) and numerically (red) for PEC substrate, fixed $h = \lambda_0/4$, and a/λ_0 varying from 0.05 to 0.1.
Right: Scattered field distribution for the $h/\lambda_0 = 1/4$. The particle is located at $\rho = z = 0$, the substrate is at $z = -h$. PEC substrate.

Comparison of the results for PEC and real substrate



Comparison of the maps of the normalized absorption rates (A/I, where I is the incoming energy flux) as a function of real and imaginary parts of permittivity for different substrates.
For both cases $h = 0.25\lambda_0$, $a = 0.07\lambda_0$, the sphere is illuminated by the perfectly absorbing beam.
Left: PEC substrate, right: aluminum substrate ($\varepsilon = -7553 + 5089i$)

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1. We have demonstrated that perfect absorption phenomenon can be achieved for a single dipole scatterer
2. We have derived the analytical expressions in dipole approximation for target permittivity and angular dependence of the perfectly absorbing beam
3. We have showed that even in case of non-PEC substrate the phenomenon of perfect absorption by a spherical particle is not destroyed.

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