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Effective electromagnetic fields of a particle situated near a substrate

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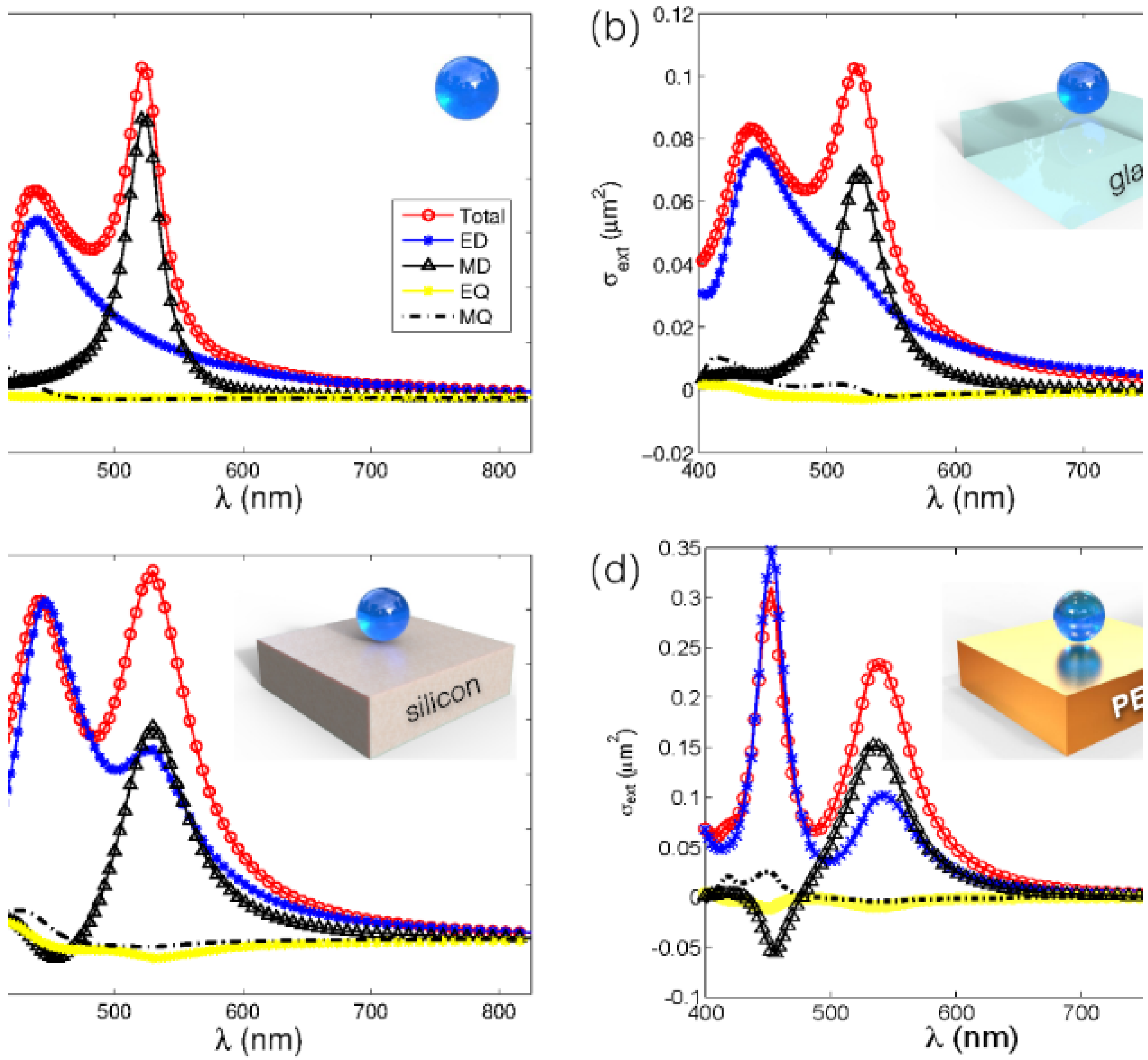
Si sphere Above a substrate: Electromagnetic coupling

- A. E. Miroshnichenko, A. B. Evlyukhin, Y. S. Kivshar, and B. N. Chichkov, *Substrate-Induced Resonant Magnetoelectric Effects for Dielectric Nanoparticles*, ACS Photonics **2**, 1423 (2015).



We start by setting the basis to find the Green tensor that describe the electric from a Cartesian multipole of an arbitrarily order. We follow similar approach to that previously introduced in Ref. [15]. Generally, such tensors can be readily found by integrating the polarization **P** over the entire scatterer volume v:

$$\mathbf{E}(\mathbf{r}) = \int_v \mathscr{G}(\mathbf{r},\mathbf{r}') \cdot \mathbf{P}(\mathbf{r}')\, d\mathbf{r}', \tag{1}$$

where **r** is a point in space, $\mathscr{G}(\mathbf{r},\mathbf{r}')$ is a Green tensor associated with the infinite background of a scatterer. The multipole decomposition of the source polarization up to the electric octapole maybe formulated as [11]

$$\begin{aligned}\mathbf{P}(\mathbf{r}') \simeq\ & \mathbf{p}\delta(\mathbf{r}'-\mathbf{r}_0) - \frac{1}{6}\mathbf{Q}\cdot\nabla\delta(\mathbf{r}'-\mathbf{r}_0) + \frac{i}{\omega}[\nabla\times\mathbf{m}\delta(\mathbf{r}'-\mathbf{r}_0)] \\ & + \frac{1}{6}\mathbf{O}\cdot[\nabla\nabla\delta(\mathbf{r}'-\mathbf{r}_0)] - \frac{i}{2\omega}[\nabla\times\mathbf{M}\nabla\delta(\mathbf{r}'-\mathbf{r}_0)] + \ldots,\end{aligned} \tag{2}$$

where $\mathbf{p},\mathbf{m},\mathbf{Q},\mathbf{M}$, $\mathbf{O}$ are respectively the electric dipole, magnetic dipole, electric quadrupole, magnetic quadrupole, and electric octapole moments. Here the Dirac delta function $\delta(\mathbf{r}'-\mathbf{r}_0)$ is expanded in a Taylor series with respect to $\mathbf{r}'$ around the origin. ∇ is the nabla operator and ω is the angular frequency. We derive only the electric octapole Green tensor since all other tensors can be found for example in Ref. [15].

One can use useful integration identity for an arbitrary scalar function $f(\mathbf{r}')$:

$$\int_v f(\mathbf{r}')\partial^m\delta(\mathbf{r}'-\mathbf{r})d\mathbf{r}' = (-1)^m\partial^m f(\mathbf{r}')|_{\mathbf{r}'=\mathbf{r}}, \tag{3}$$

and to find the electric field scattered by electric octapole source, we substitute the relevant term from eq 2 into eq 1 :

Example: The Electric Octapole Field

ıply the previous integration identity. After some simple math, the Electric octapole elements can be foun

$$\mathbf{E}^{O}(\mathbf{r}) = \mathscr{G}^{EO}(\mathbf{r}) : \mathbf{O},$$

ıe green function terms defined as

$$g^{EO}_{ij\alpha\beta}(\mathbf{r}) = \frac{k^2}{6\varepsilon_0}\partial_\alpha\partial_\beta\gamma_{ij}g_0(\mathbf{r}),$$

$\gamma_{ij} = (\partial_i\partial_j - k^2\delta_{ij})$ and $g_0(\mathbf{r})$ is the scalar Green function.where subscript indices denote component $,y,z)$. Now after generalizing the process to finding the Greens tensor of any order, we focus on studying evolution near a substrate in the next section.

GREEN TENSOR DECOMPOSITION

In a first approach we classify the multipoles in accordance to their contributions to the electric fields and high orders terms of field decomposition(as depicted in Eq.2). In accordance every Green tensor is decomposed into two tensors, one that scatter perpendicular to the plane of the substrate and another scatter in parallel. In this section we explicitly explain the scenario for tensors that describe the electric field in space and also the electric field gradient. However the method can be easily generalized to any high order terms of field derivatives. We transform the coordinate from the Cartesian xyz to the spv coordinate systems as follow [16]:

$$\begin{aligned}
\mathbf{E}^{r}\left(\mathbf{r},\mathbf{r}_{sub}\right) &= \left(\mathbf{e}_{s}^{-}E_{s}^{r}+\mathbf{e}_{p}^{+}E_{p}^{r}\right)e^{i\mathbf{k}\cdot\left(\mathbf{r}-\mathbf{r}_{sub}\right)} \\
&= r_{s}\mathbf{e}_{s}^{-}\left(\mathbf{e}_{s}^{-}\cdot\mathscr{G}^{(-),E}\left(\mathbf{r},\mathbf{r}_{sub}\right)\cdot\mathscr{P}\right)+r_{p}\mathbf{e}_{p}^{+}\left(\mathbf{e}_{p}^{-}\cdot\mathscr{G}^{(-),E}\left(\mathbf{r},\mathbf{r}_{sub}\right)\cdot\mathscr{P}\right), \\
&= \left(r_{s}\mathscr{G}^{E,r,s}\left(\mathbf{r},\mathbf{r}_{sub}\right)+r_{p}\mathscr{G}^{E,r,p}\left(\mathbf{r},\mathbf{r}_{sub}\right)\right)\cdot\mathscr{P}
\end{aligned} \tag{7}$$

and

$$g_{ijk\ldots}^{E,r,n}\left(\mathbf{r},\mathbf{r}_{sub}\right)=e_{ni}^{+}e_{n\gamma}^{-}g_{\gamma jk\ldots}^{(-),E}\left(\mathbf{r},\mathbf{r}_{sub}\right)e^{2i\mathbf{k}\cdot\mathbf{r}_{\mathrm{sub}}}, \tag{8}$$

where $\mathbf{E}^{r}$ is the field reflected by the substrate. $n=s,p$ and $\mathscr{P}$ represents multipole moments. $\mathbf{e}_{s}$ and $\mathbf{e}_{p}$ are the unit vectors perpendicular and parallel a plane containing the normal to the substrate with the superscripts $\mp$ are determine

Transmitted Components

$$\mathbf{E}^t(\mathbf{r},\mathbf{r}_{sub}) = \left(\mathbf{e}_s^- E_s^t + \mathbf{e}_p^+ E_p^t\right) e^{i\mathbf{k}\cdot(\mathbf{r}-\mathbf{r}_{sub})}$$

$$= t_s\mathbf{e}_s^-\left(\mathbf{e}_s^-\cdot\mathscr{G}^{(-),E}(\mathbf{r},\mathbf{r}_{sub})\cdot\mathscr{P}\right) + t_p\mathbf{e}_p^-\left(\mathbf{e}_p^-\cdot\mathscr{G}^{(-),E}(\mathbf{r},\mathbf{r}_{sub})\cdot\mathscr{P}\right) \quad (9)$$

$$= \left(t_s\mathscr{G}^{E,t,s}(\mathbf{r},\mathbf{r}_{sub}) + t_p\mathscr{G}^{E,t,p}(\mathbf{r},\mathbf{r}_{sub})\right)\cdot\mathscr{P}$$

with the Green tensors terms

$$g^{E,t,n}_{ijk\ldots}(\mathbf{r},\mathbf{r}_{sub}) = e^-_{ni}e^-_{n\gamma}g^{(-),E}_{\gamma jk\ldots}(\mathbf{r},\mathbf{r}_{sub})\,e^{2i\mathbf{k}\cdot\mathbf{r}_{\mathrm{sub}}} \quad (10)$$

where the $\mathbf{E}^t$ is the transmitted field; t_s and t_p the Fresnel transmission coefficients. Worth noticing, though, the differences in Eq. 8 and 11 in defining the basis for reflected and transmitted outgoing fields. At last, we repeat the steps Eqs.7-8 to find the reflected fields gradient

$$\mathbf{F}^r(\mathbf{r},\mathbf{r}_{sub}) = \frac{1}{2}\nabla\mathbf{E}^r(\mathbf{r},\mathbf{r}_{sub}) + \frac{1}{2}\left(\nabla\mathbf{E}^r(\mathbf{r},\mathbf{r}_{sub})\right)^T \quad (11)$$

Final Step: the Green tensor for the reflected Octapole field gradient

where $\mathbf{F}^r$ is the reflected field gradient tensor. And the Green tensors that describe field gradient scattering may hav
the following form

$$g_{ijkl}^{r,(s,p)}(\mathbf{r},\mathbf{r}_{sub},\mathbf{r}_p) = \frac{1}{2}\left(\partial_i \mathbf{e}_j^+ + \partial_j \mathbf{e}_i^+\right)\mathbf{e}_\gamma^- g_{\gamma kl}^{(-)}(\mathbf{r}_{sub},\mathbf{r}_p)\, e^{i\mathbf{k}^+\cdot(\mathbf{r}-\mathbf{r}_{sub})}. \quad (12$$

We have explained the method to decompose any Green tensor into a summation of two tensors describe perpen
dicular fields. We then applied Fresnel coefficients to these tensors to explicitly account for the reflected fields fror
the substrate and the transmitted ones.

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