Periodic photonic structures are one of the most important components in modern photonics since they form the basis of optical elements and devices. Such structures can be photonic crystals [1] and metamaterials [2].

In the periodic structure, Bragg and Mi resonances can occur, and if any of the Mi resonances are lower in photon energy than the Bragg resonance, then we are dealing with a metamaterial.

References
Goal of theoretical work

At a large dielectric constant, the TE\textsuperscript{11} resonance is observed lower in photon energy than the Bragg resonance, which means that there is a quadrupole-driven metamaterial in a certain frequency range. Thus, the aim of this work is to describe the quadrupole-driven metamaterial.

References

Dipole metamaterials have been studied for a long time, and works have been written that describe these materials in various ways: Rigorous coupled-wave analysis (RCWA) + discrete dipole approximation (DDA) [4] (“coins”) or, for example, metamaterials composed of sparse high-index dielectric rods allow a homogenization procedure by using the approach proposed in work [5]

**References**


Let's choose a frequency range where quadrupoles will make the greatest contribution to the scattered field. The field created by the quadrupole can be calculated as:

\[
E = \hat{G}^{eq} \hat{Q}
\]

where \(\hat{G}^{eq}\) is the quadrupole Green's function, the tensor of the third rank, and \(\hat{Q}\) is the quadrupole tensor, which is defined as:

\[
Q_{ij}^{eq} = \alpha_{ij}^{eq} (\partial_i E_j + \partial_j E_i)
\]

where \(\alpha_{ij}^{eq}\) is the quadrupole polarizability of a single cylinder.
since all quadrupoles create a field that acts on others

\[ E_{k}^{bg} = E_{k}^{0} + \sum_{l \neq k} G^{eq}(r_{k}, r_{l}) \hat{Q}_{k} e^{i k_{j}(r_{l} - r_{k})} \]

using the Bloch theorem and introducing the notation

\[ E_{k}^{bg} = E_{k}^{0} + \hat{C}^{eq}(k_{||}) \alpha^{eq} \left( \partial_{i} E_{j, k}^{bg} + \partial_{j} E_{i, k}^{bg} \right) \]

introducing the effective polarizability in the form

\[ Q_{eij, k}^{eq} = \alpha_{eff}^{eq} \left( \partial_{i} E_{j, k}^{0} + \partial_{j} E_{i, k}^{0} \right) \]

we can finally express it

\[ \alpha_{eff}^{eq} = \alpha^{eq} \left( \hat{I} - \hat{C}^{eq}(k_{||}) \alpha^{eq} \left( \partial_{i} \delta_{j} + \partial_{j} \delta_{i} \right) \right)^{-1} \]

Quadrupole polarizability on a single particle is proposed to be calculated using the software package COMSOL Multiphysics.
Calculations: $\varepsilon = 80$

Experiment:

Work in progress
1. The basis for the construction of the theory of quadrupole-driven metamaterials has been prepared;

2. It is necessary to numerically find the value of the polarizability of a single cylinder, find the effective polarizability;

3. And finally, compare the results that we will get in the future with the experiment.