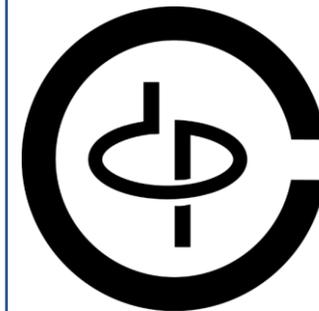


# Spontaneous Symmetry Breaking and Control of the Radiation from Microlaser Arrays

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The effect of spontaneous symmetry breaking is well known in the dynamics of the nonlinear waves. The essence of the effect is that, although a symmetric system always possesses a symmetric solution, this solution must not necessarily be stable. The instability of the symmetric state results in the switching of the system to another state that can be asymmetric. In nonlinear optics this effect was first revealed in two-beam nonlinear Fabry-Perot interferometer [1], see Fig.1a, and later was widely studied in many optical systems [2,3].

This effect is highly important in optical systems which can provide so-called “Bound State in the Continuum” (or “BIC”) [4,5]. BIC is a special high-Q state, which coexist with continuous spectrum of radiating waves, but remains localized. Such unusual behavior can be explained by destructive interference of leaky modes in

## References

- [1] Haelterman and P. Mandel, *Opt. Lett.* 15, 1412 (1990);
- [2] N. Akhmediev and A. Ankiewicz, *Phys. Rev. Lett.* 70, 2395 (1993);
- [3] B. A. Malomed, *Spontaneous Symmetry Breaking, Self-Trapping, and Josephson Oscillations* (Springer, Berlin, Heidelberg, 2013);
- [4] C. W. Hsu, *et al*, *Nat. Rev. Mater.* 1 (2016);
- [5] D. C. Marinica, *et al*, *Phys. Rev. Lett.* 100, 183902, 2008;
- [6] S. D. Krasikov, *et al*, *Phys. Rev. B* 97(2018).

the system, see Fig. 1b. Therefore, BIC is uncoupled with incoming pumping radiation and can not be excited directly. But because of the nonlinearity and symmetry breaking bifurcation the parametric excitation of BIC can take place. As a result a quasi-BIC appears, which is still high-Q [6].

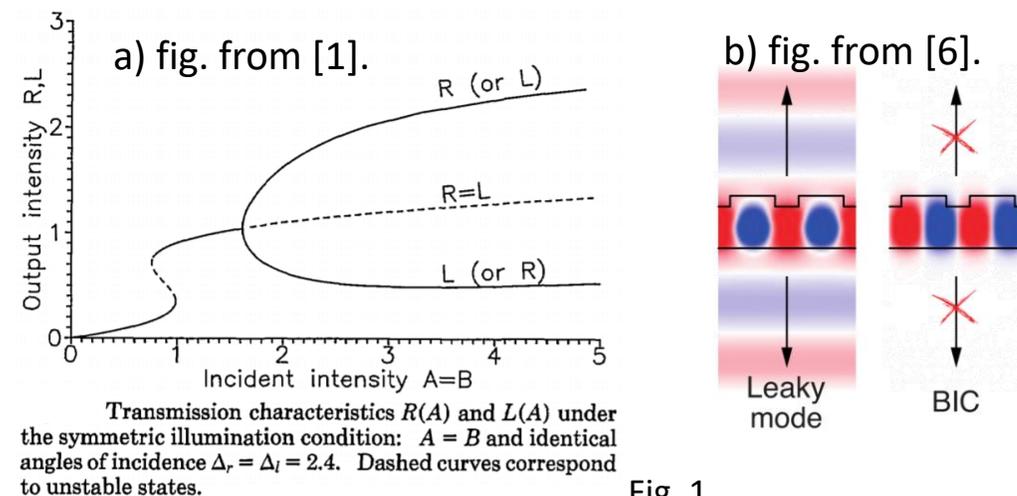
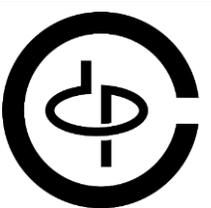
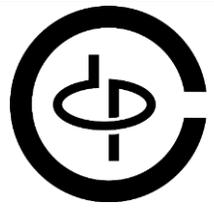


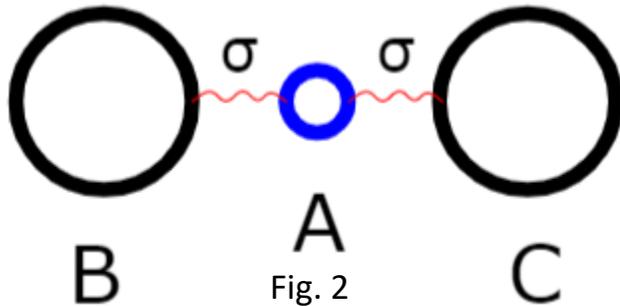
Fig. 1

In our work we consider the effect of spontaneous symmetry breaking in laser system with BIC and investigate the influence of this effect on radiation from laser arrays.



## Considered microlaser system

To begin with, in our work we considered a simple system consisting of two identical single-mode active nonlinear resonators (schematically shown as “B” and “C” in a figure 1) interacting through rescattering on an additional passive linear resonator (shown as “A”). We will call such laser system as <<trimer>>.



The laser pair is pumped by external pumping and the radiation from each of the nonlinear resonators is partially transferred to the central resonator.

## Mathematical model

Using tight-binding approximation the complex amplitudes of field in the resonators can be described mathematically by following equations:

$$\begin{cases} \partial_t B = \Gamma B - \beta |B|^2 B + i\alpha |B|^2 B + i\delta B + i\sigma A \\ \partial_t C = \Gamma C - \beta |C|^2 C + i\alpha |C|^2 C + i\delta C + i\sigma A \\ \partial_t A = -\gamma A + i\sigma(B + C), \end{cases}$$

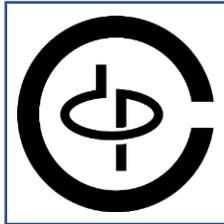
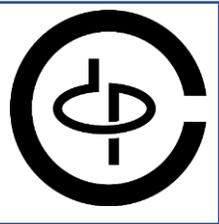
where  $B$  и  $C$  are complex amplitudes of the field in active resonators,  $A$  is amplitude in linear resonator,  $\Gamma$  is gain of active medium,  $\beta$  is nonlinear losses coefficient,  $\alpha$  is Kerr nonlinearity coefficient,  $\delta$  is frequency detuning from resonant frequency of the central resonator,  $\sigma$  is conservative coupling of the active pair with central resonator,  $\gamma$  is losses of the linear resonator.

These equations can be rewritten in terms of “bright” and “dark” modes, where

$X = 1/2 (B + C)$  is “bright” mode and  $Y = 1/2 (B - C)$  is “dark” (or BIC) mode:

$$\begin{cases} \partial_t X = \Gamma X - (\beta - i\alpha)(KX + MY) + i\delta X + i\sigma A \\ \partial_t Y = \Gamma Y - (\beta - i\alpha)(KY + M^*X) + i\delta C \\ \partial_t A = -\gamma A + i\sigma X, \end{cases}$$

where  $K = 2|X|^2 + |Y|^2$  and  $M = 2X^*Y$ .



## Simple stationary states

The simplest stationary states are states corresponding to noninteracting dark and bright modes.

Dark state. Amplitudes and frequency of dark states can be easily found analytically:

$$B = -C, \quad A = 0;$$

$$B = |\rho|e^{i\omega t};$$

$$|\rho| = \sqrt{\Gamma/\beta}; \quad \omega = \alpha\Gamma/\beta + \delta.$$

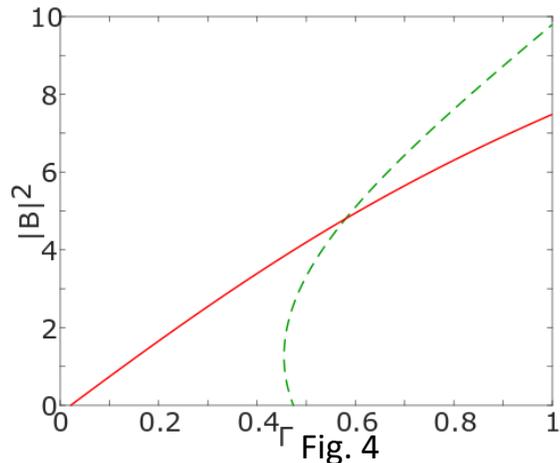
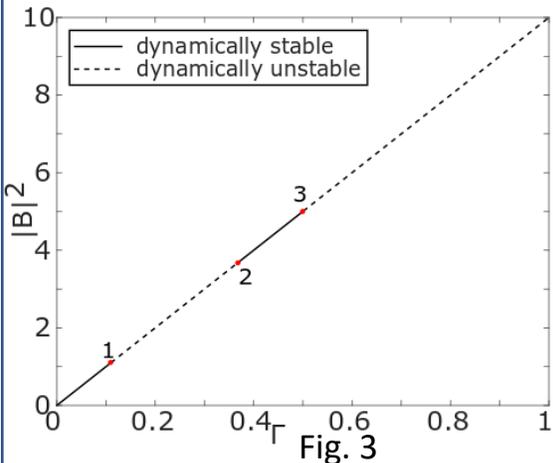
Bright state. Amplitudes and frequency of bright states also can be found analytically:

$$B = C = |\rho|e^{i\omega t}, \quad A \neq \tilde{A}e^{i\omega t};$$

$$|\rho| = \sqrt{\Gamma/\beta - 2\sigma^2\gamma/[\beta(\gamma^2 + \omega^2)]};$$

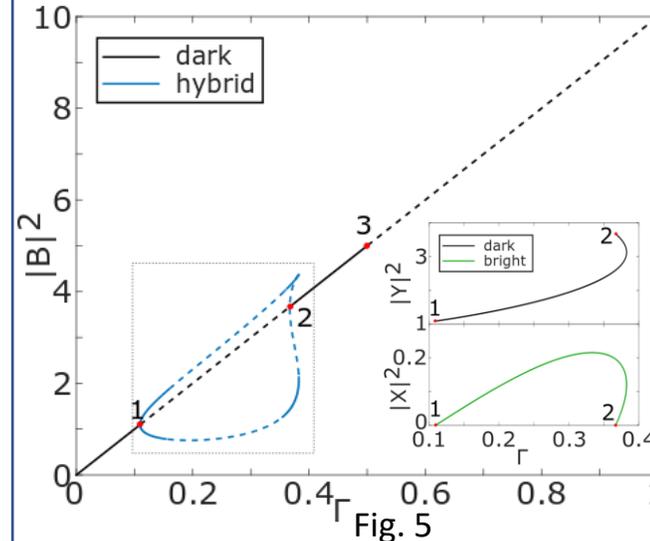
$$\tilde{A} = 2i\sigma|\rho|/[i\omega + \gamma];$$

$$\omega = \alpha|\rho|^2 + \delta + 2\sigma^2\omega/[\gamma^2 + \omega^2].$$

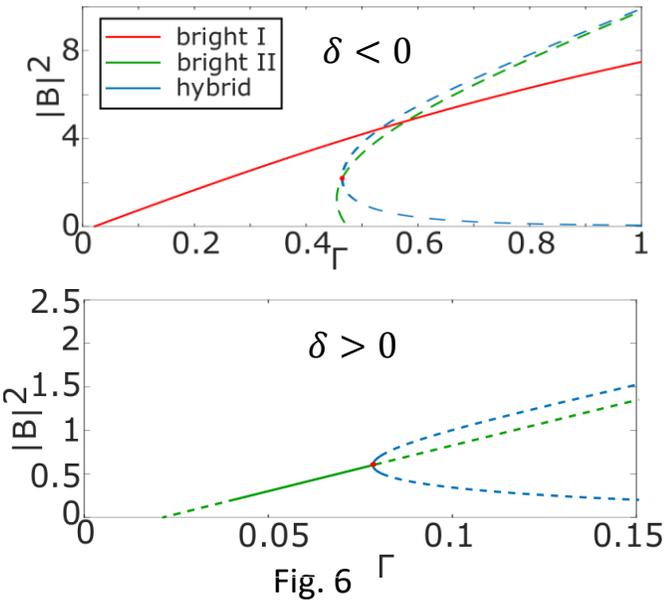


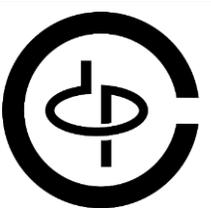
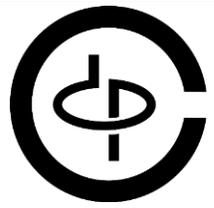
## Hybrid states

It happened that the *dark* states can be unstable with respect to the perturbations having a structure of the *bright* mode. This instability can result in the formation of the hybrid states having both the bright and the dark components (as it is demonstrated in the inset). This is a symmetry breaking bifurcation.



The opposite situation is also possible, when one of the *bright* states parametrically excites a *dark* mode, and new hybrid state appears, but it is mostly unstable.





## Interaction of the laser trimer with a waveguide

We have considered the radiation of such a laser trimer in a discrete waveguide. It is clear that if trimer is in the dark state, there is no radiation. But if the trimer is in bright or hybrid state, a part of the radiation transfers to the waveguide, where depending on frequency it can either propagate or decay.

Mathematically the system can be described by following equations:

$$\begin{cases} \partial_t B_i = \Gamma B_i - \beta |B_i|^2 B_i + i\alpha |B_i|^2 B_i + i\delta B_i + i\sigma A_i + \tilde{\sigma} D_{m_i} \\ \partial_t C_i = \Gamma C_i - \beta |C_i|^2 C_i + i\alpha |C_i|^2 C_i + i\delta C_i + i\sigma A_i + \tilde{\sigma} D_{m_i} \\ \partial_t A_i = -\gamma A_i + i\sigma(B_i + C_i), \\ \partial_t D_n = -\tilde{\gamma} D_n + i\tilde{\sigma}(B_i + C_i)\delta_{n,m_i} + i\sigma'(D_{n+1} + D_{n-1} - 2D_n), \end{cases}$$

where  $D_n$  is complex amplitude of field in  $n$ -element of the discrete waveguide,  $\tilde{\sigma}$  is coupling between laser pair with waveguide,  $\tilde{\gamma}$  is losses in waveguide and  $\sigma'$  is coupling between elements of the waveguide.

Dispersion of linear waves in the waveguide:  $\Omega = 2\sigma'(\cos q - 1)$

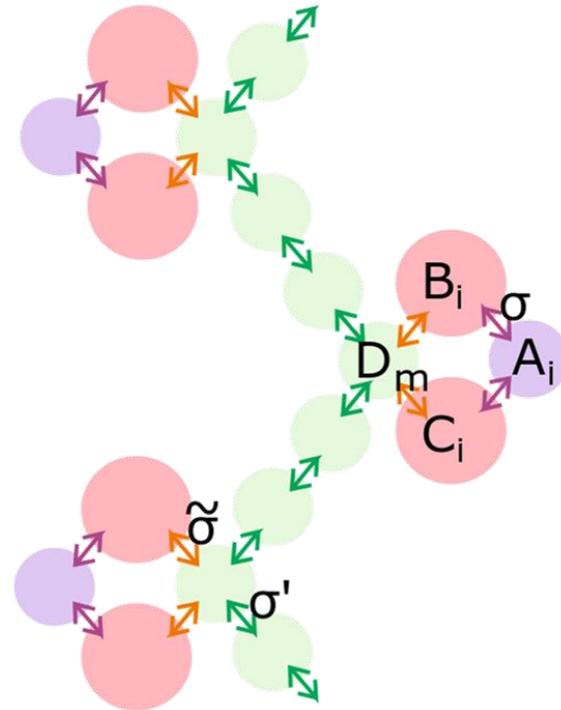
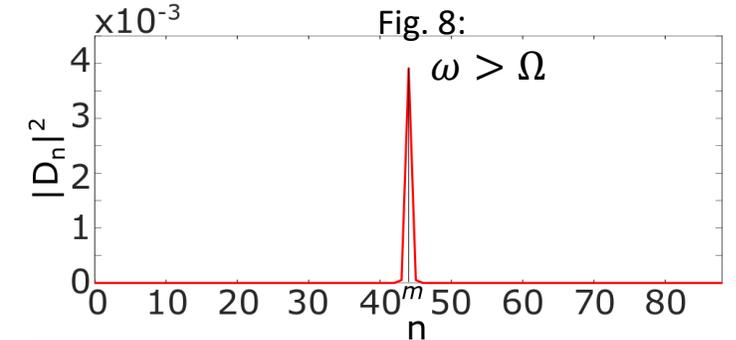
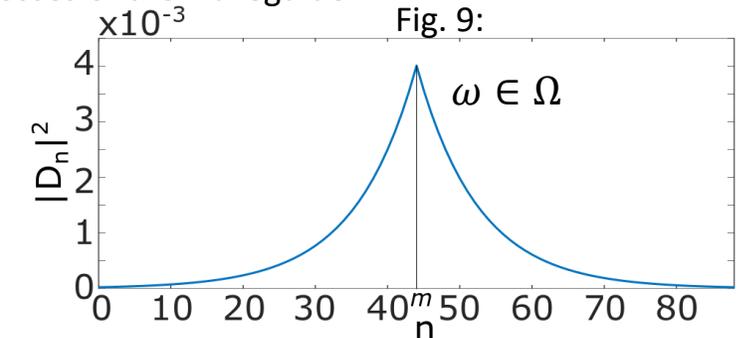


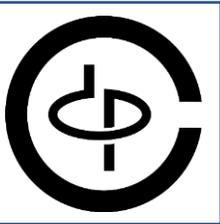
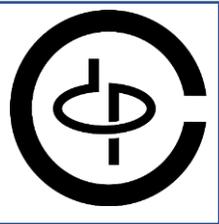
Fig. 7

If frequency  $\omega$  is out of dispersion relation the radiation is localized near the trimer:



In opposite case, the radiation propagates through the waveguide and fades only because of the losses of the waveguide:





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## Phase synchronization of laser trimers

If several trimers are interacting through “tails” of evanescent field the effect of phase synchronization of all trimers takes place. The phase difference become either “0” or  $\pi$ , depending on the distance between the trimers.

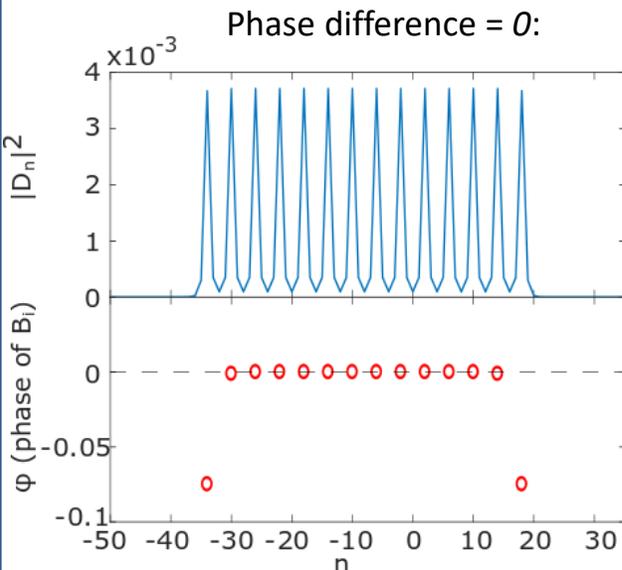


Fig. 10

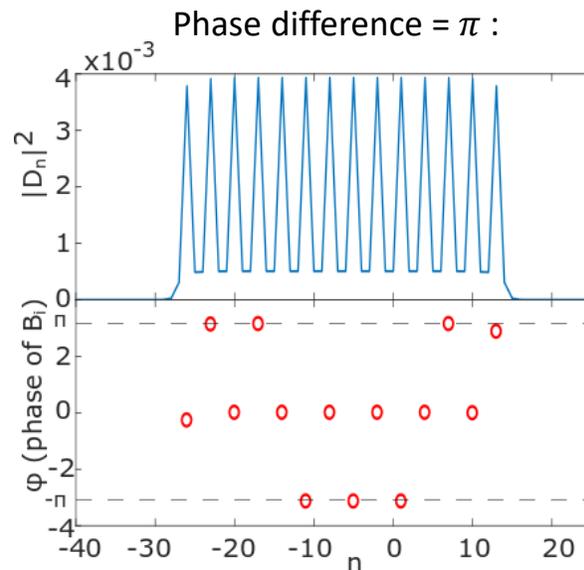


Fig. 11

Of course, if a distance between trimers is large the synchronization is not possible. The maximum distance depends on the detuning from the waveguide resonant frequencies.

## Symmetry breaking of the radiation

If trimers are interacting through propagating waves, they can form different types of radiation patterns. If distance between trimers is greater than a wavelength the radiation pattern is symmetric, but in opposite case a pattern with broken symmetry can appear. In the later case, most the radiation propagates only in one direction.

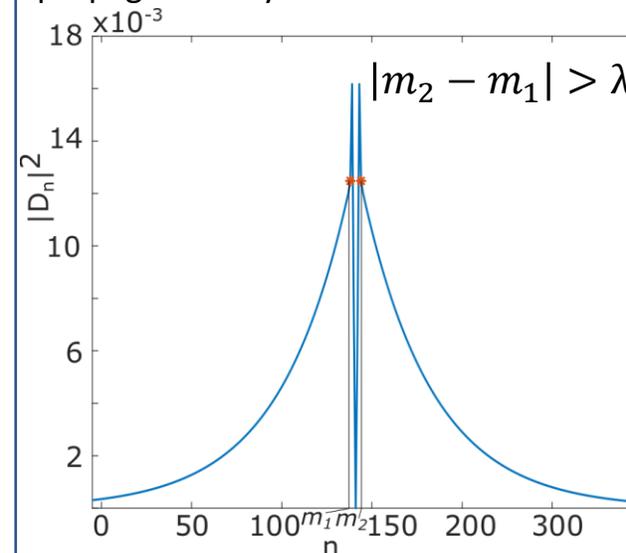


Fig. 12

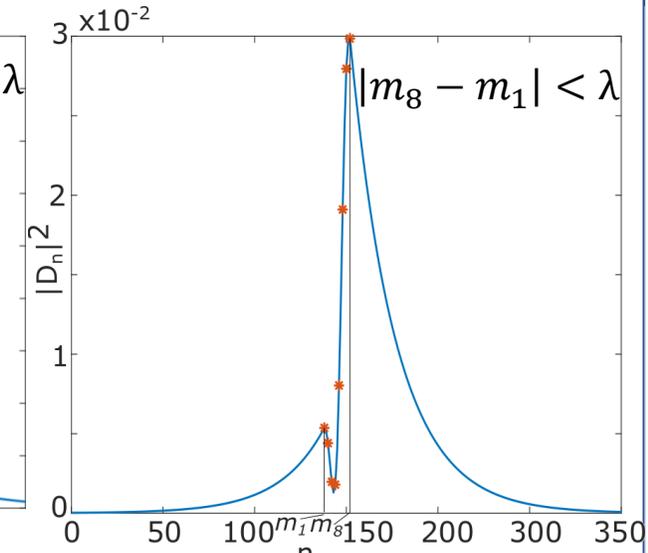
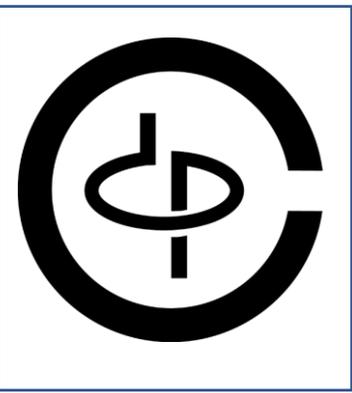
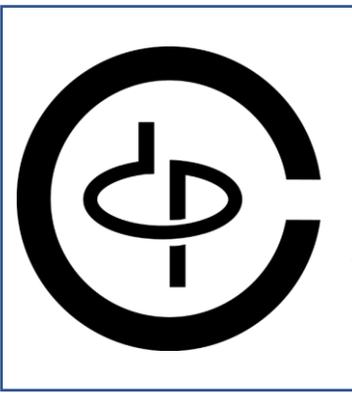


Fig. 13

The direction of the radiation can be easily controlled by insignificant symmetry breaking of the pump.



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1. The dynamics of active nonlinear resonators interacting through rescattering on the linear coupler (or <<trimers>>) has been studied, and it has been shown that they can form nonradiative (or “dark”), radiative (or “bright”), and hybrid states with broken symmetry, each of which can be dynamically stable.
2. The interaction of several trimers through a waveguide can result in phase synchronization of the trimers. In this case phase difference between neighbor trimers is 0 or  $\pi$ .
3. The interaction of trimers can lead to additional symmetry breaking. As a result, the radiation from trimers can propagate mostly in one direction of the waveguide.

The described effects can be used to obtain controllable directed radiation.

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