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Tuning 2ND AND 3RD Order Exceptional Points by Kerr-Nonlinearity

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- The exceptional point (EP) is a degeneracy in non-Hermitian systems at which the eigenvectors become parallel.

- It is different from degeneracy in Hermitian systems where the eigenvectors are orthogonal.

- The abrupt phase transitions around this point in photonic systems leads to exotic functionalities such as unidirectional invisibility, laser mode selectivity and sensitivity enhancement.

- Although EP is introduced in Quantum Mechanics, but it can be observed in optics and photonics containing resonators with gain and loss.

Analogous between Nonlinear Shrodinger Equation in Quantum Mechanics and Nonlinear Coupled Mode Approach in Optics

nonlinear Schrodinger equation (NLSE) or Gross–Pitaevskii equation

$$\left(-\frac{\hbar^2}{2M}\frac{\partial^2}{\partial x^2} + V(x) + g|\psi(x,t)|^2\right)\psi(x,t) = i\hbar\frac{\partial\psi(x,t)}{\partial t}$$

Applying on a two level system

$$\begin{pmatrix} \varepsilon - 2i\gamma + g|\psi_1|^2 & \upsilon \\ \upsilon & -\varepsilon + g|\psi_2|^2 \end{pmatrix}\begin{pmatrix}\psi_1 \\ \psi_2\end{pmatrix} = i\hbar\frac{\partial}{\partial t}\begin{pmatrix}\psi_1 \\ \psi_2\end{pmatrix}$$

Time-Independent Form

$$\begin{pmatrix} \varepsilon - 2i\gamma + g|\psi_1|^2 & \upsilon \\ \upsilon & -\varepsilon + g|\psi_2|^2 \end{pmatrix}\begin{pmatrix}\psi_1 \\ \psi_2\end{pmatrix} = \mu\begin{pmatrix}\psi_1 \\ \psi_2\end{pmatrix}$$

Coupled-Mode Theory with Kerr-nonlinearity

$$\frac{d\psi_1}{dt} = (-i\omega_1 - \gamma_1 - i\,g|\psi_1|^2)\psi_1 - i\upsilon\psi_2$$

$$\frac{d\psi_2}{dt} = (-i\omega_2 - \gamma_2 - i\,g|\psi_2|^2)\psi_2 - i\upsilon\psi_1$$

Assuming monochromatic excitation

$$\frac{d}{dt} \rightarrow -i\omega$$

$$\begin{pmatrix} \omega_1 - i\gamma_1 + g|\psi_1|^2 & \upsilon \\ \upsilon & \omega_2 - i\gamma_2 + g|\psi_2|^2 \end{pmatrix}\begin{pmatrix}\psi_1 \\ \psi_2\end{pmatrix} = \omega\begin{pmatrix}\psi_1 \\ \psi_2\end{pmatrix}$$

Matrix Form of Nonlinear Coupled Mode Theory

The matrix equation, describes a nonlinear non-Hermition system

$$\begin{bmatrix} \varepsilon + g|a|^2 - 2i\delta & \upsilon \\ \upsilon & -\varepsilon + g|b|^2 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \mu \begin{bmatrix} a \\ b \end{bmatrix}$$

shift the energy levels of the Hamiltonian by

$$-\frac{g}{2}\left(|a|^2 + |b|^2\right)$$

$$\begin{bmatrix} \varepsilon + c\kappa - 2i\delta & \upsilon \\ \upsilon & -\varepsilon - c\kappa \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \mu \begin{bmatrix} a \\ b \end{bmatrix} \qquad \begin{matrix} c = g/2 \\ \kappa = |a|^2 - |b|^2 \end{matrix}$$

After some algebraic manipulation (Stokes parameters)

$$(g^2 + \rho^2)\kappa^4 + 2gh\kappa^3 + (1 + h^2 - \rho^2 - g^2)\kappa^2 - 2gh\kappa - h^2 = 0$$

$$g = \frac{c}{\upsilon}, \; h = \frac{\varepsilon}{\upsilon}, \; \rho = \frac{\gamma}{\upsilon}$$

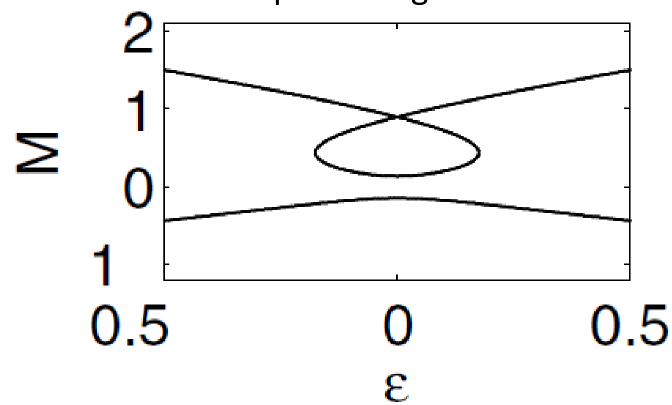
There can be two up to four real roots and each of them is connected to a complex eigenvalue by

$$\mu = c + \frac{\varepsilon}{\kappa} - i\gamma(1 + \kappa)$$

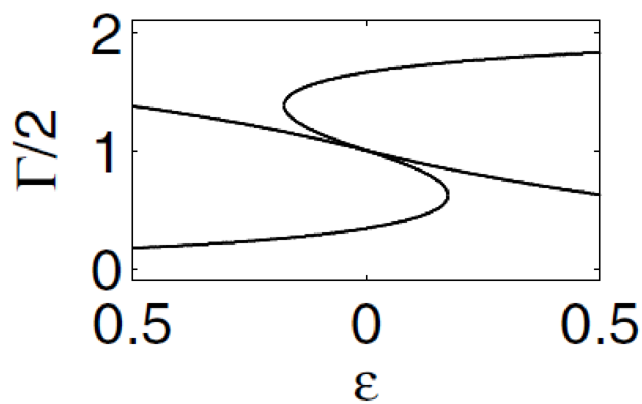
The coupling and dissipation factors are unequal

$$\nu = 1.01 > \gamma$$

Real part of eigenvalues



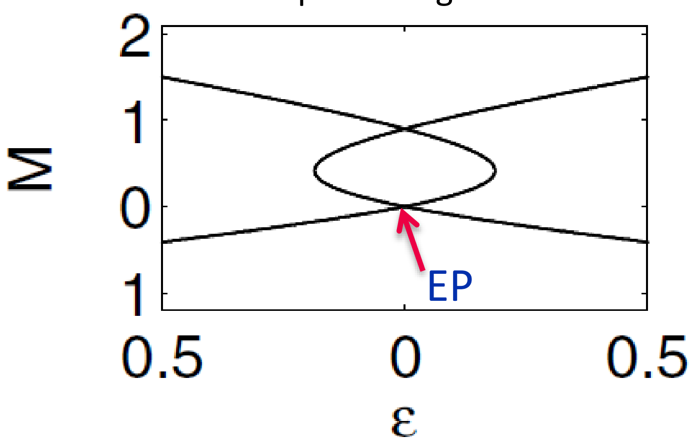
Imaginery part of eigenvalues



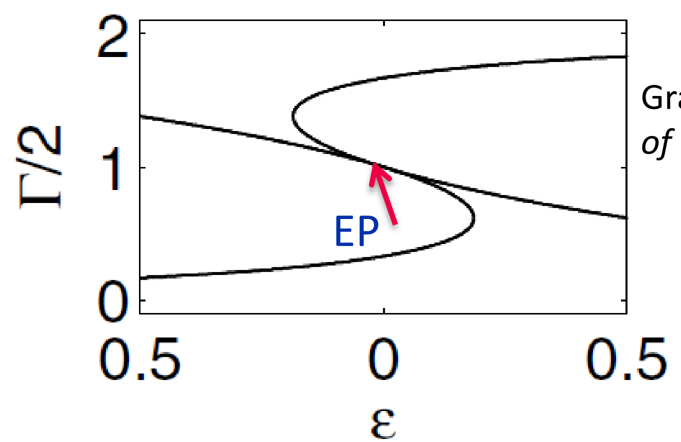
The coupling and dissipation factors are equal

$$\nu = 1.00 = \gamma$$

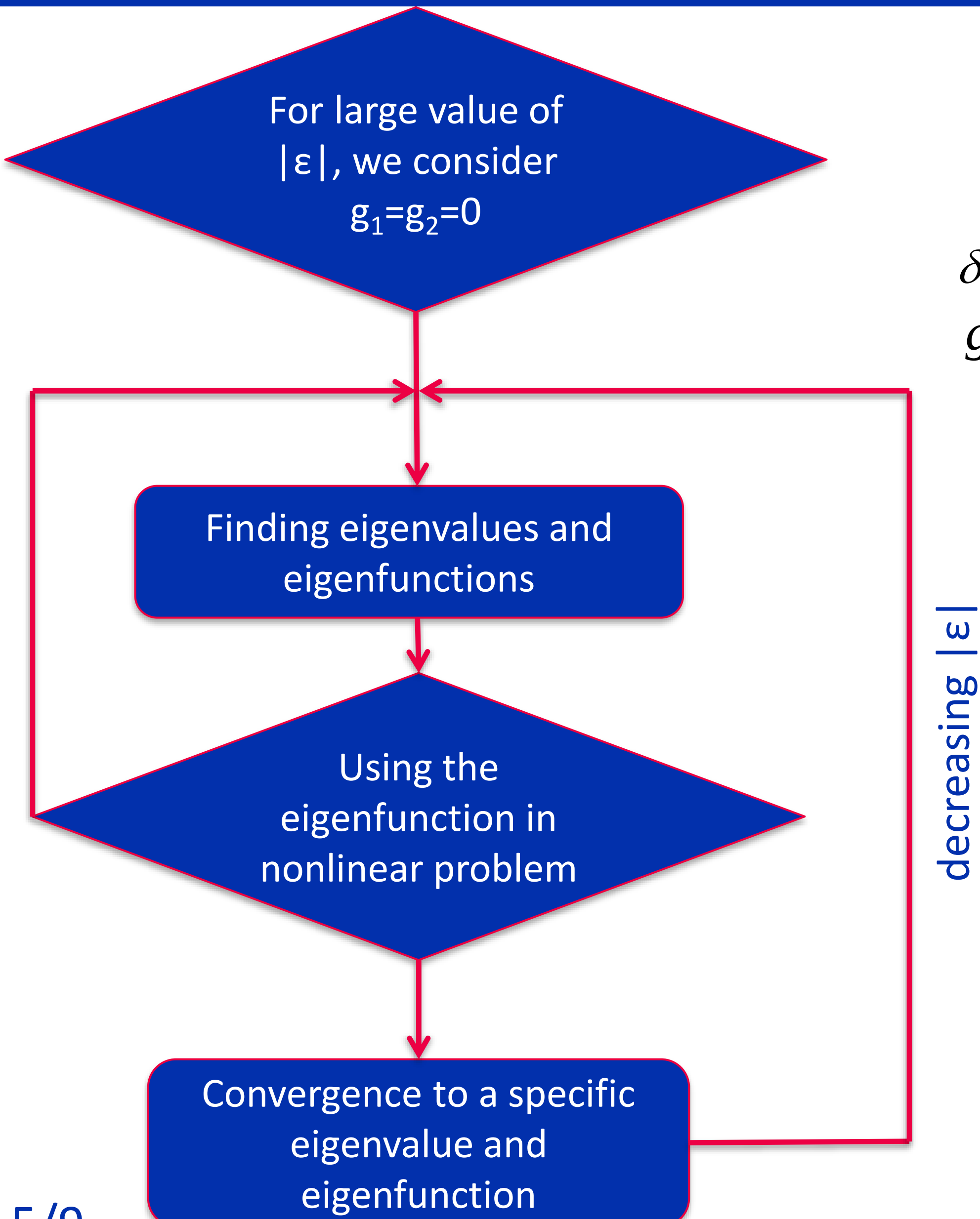
Real part of eigenvalues



Imaginery part of eigenvalues



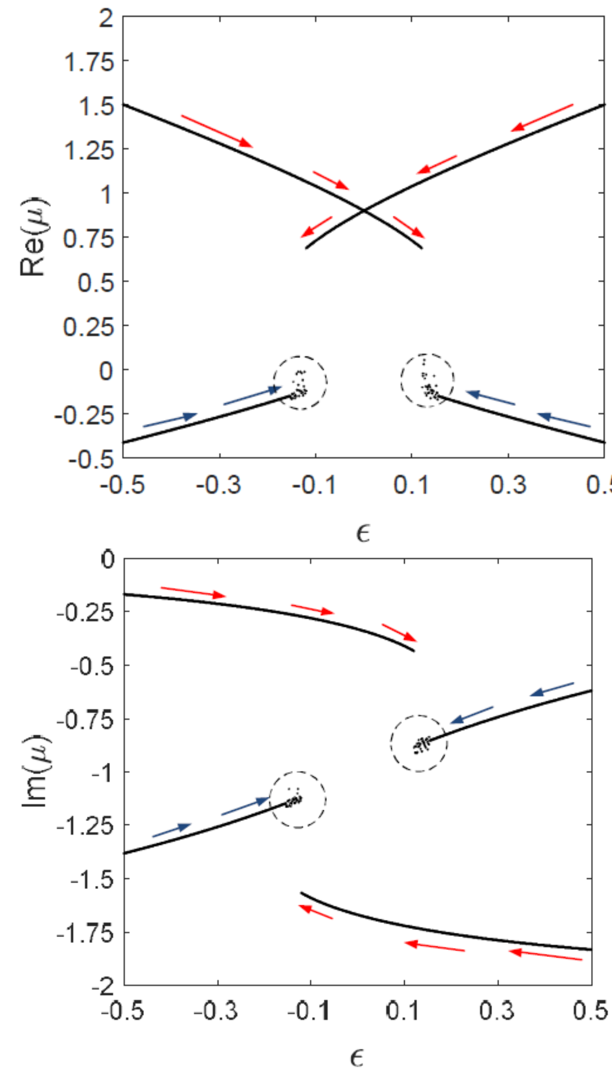
Graefe, *Czechoslovak Journal of Physics*, 2006



$$\begin{bmatrix} \varepsilon + g_1|a|^2 - 2i\delta & \upsilon \\ \upsilon & -\varepsilon + g_2|b|^2 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = \mu \begin{bmatrix} a \\ b \end{bmatrix}$$

$$\delta = \upsilon = 1$$

$$g_1 = g_2 = 1.8$$

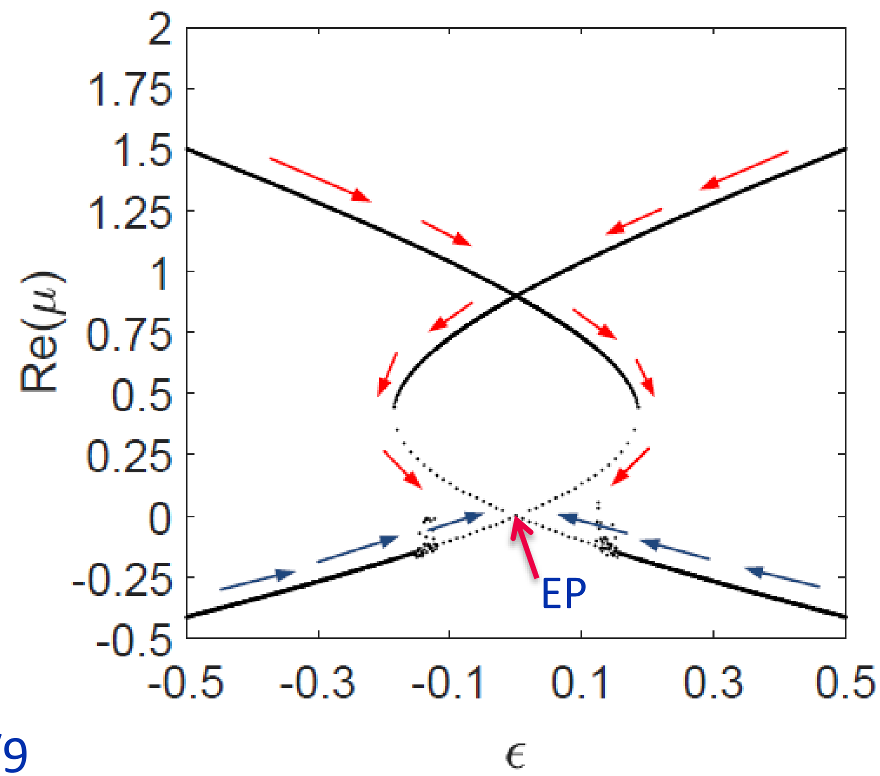


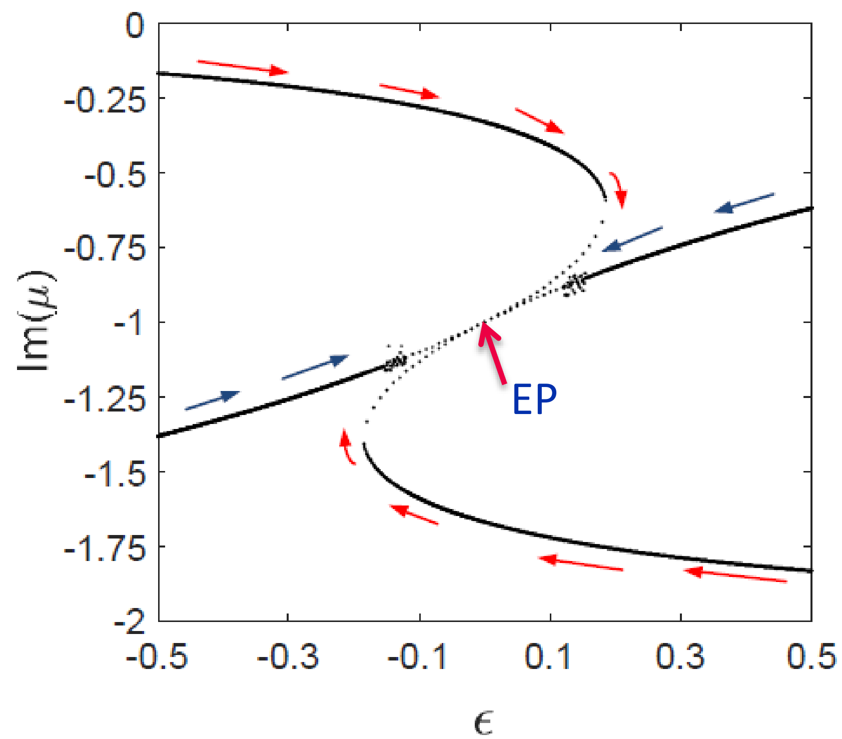
At the discontinuities, we consider the eigenfunction

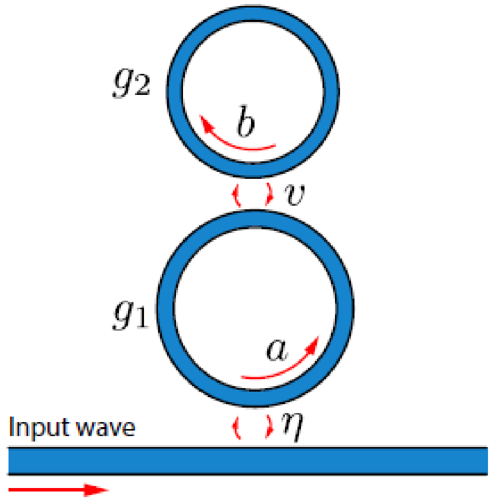
$$(a_c + \Delta_1 + i\Delta_2, b_c + \Delta_3 + i\Delta_4)$$

as initial value of the next step, and change the $\Delta_1, \Delta_2, \Delta_3, \Delta_4$

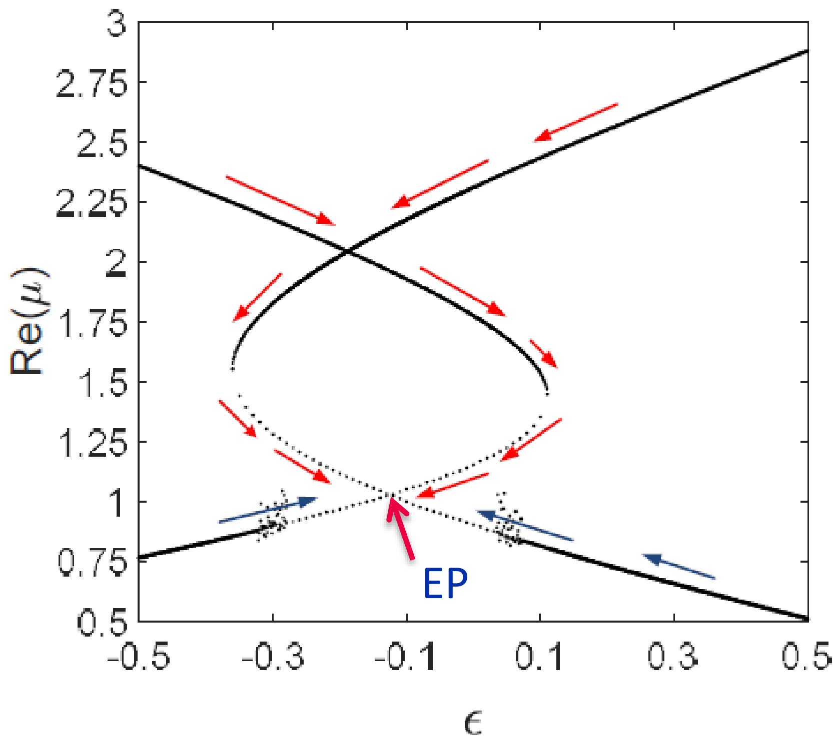
with considering the changing behavior of eigenfunctions in the previous steps.

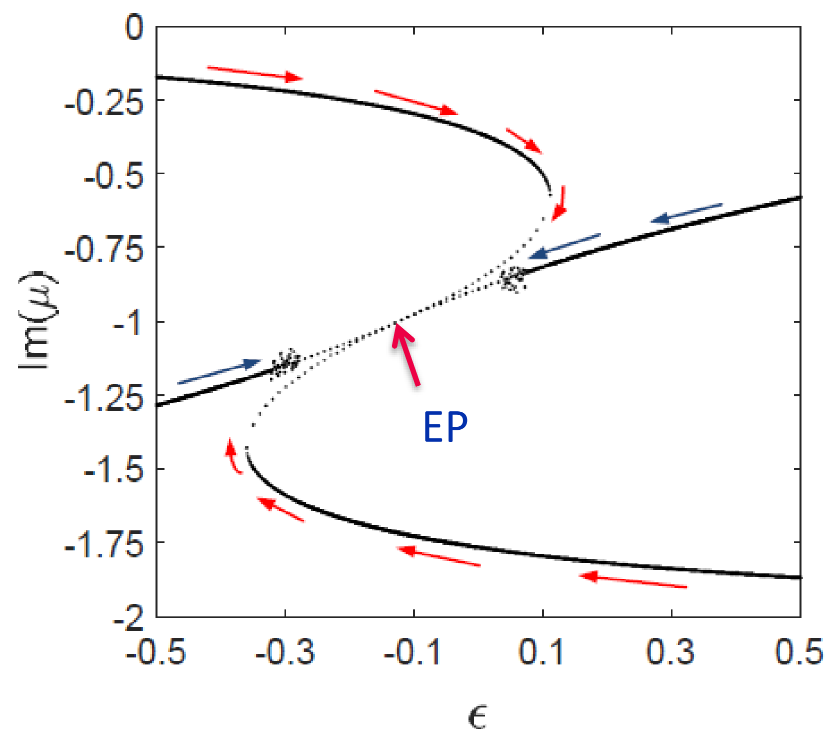






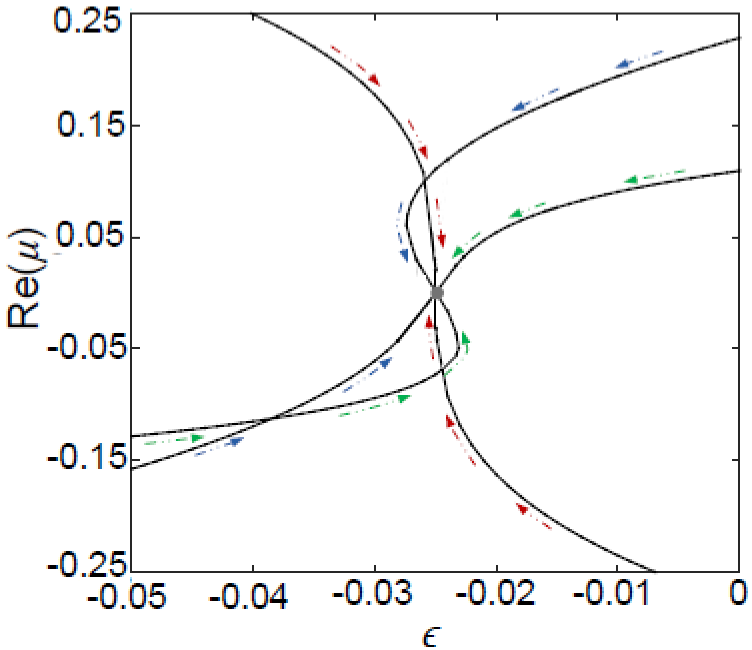
$g_1 = 2.3,\ g_2 = 1.8$

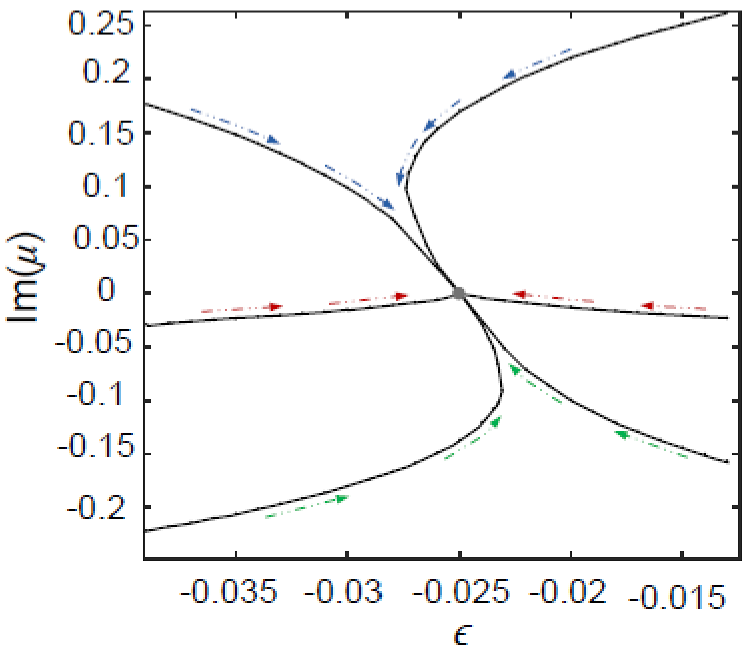




$$\begin{bmatrix} i\gamma+\varepsilon+g_1|a|^2 & \kappa & 0 \\ \kappa & g_2|b|^2 & \kappa \\ 0 & \kappa & -i\gamma+g_3|c|^2 \end{bmatrix}\begin{bmatrix} a \\ b \\ c \end{bmatrix}=\mu\begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

$$\gamma=\sqrt{2}\kappa \qquad \kappa=1 \qquad g_1=0.1, g_2=g_2=0$$





Conclusion

- We proposed a numerical method based on SFC and iteration methods to solve nonlinear non-Hermitian eigenvalue problems. This method is performed in two stages.

- It is observed that both 2nd and 3rd order EP can be tuned by the contrast between the Kerr nonlinearities in the matrix equation.

Thank you